Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (CHE)/SEM-1/CHE-04/2011-12

2011
ADVANCED NUMERICAL METHODS IN CHEMICAL ENGINEERING

Time Allotted : 3 Hours   *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions taking at least *one* from each group.

Group - I

1. Use the Fourth-order Runge-Kutta method to solve the following set of differential equations assuming that at $x = 0$, $y_1 = 4$ and $y_2 = 6$ Integrate to $x = 2$ with a step size of 0·5.

$$\frac{dy_1}{dx} = -0.5y_1; \quad \frac{dy_2}{dx} = 4 - 0.3y_2 - 0.1y_1 \qquad 14$$

2. Obtain the eigenvalues of the following matrix

   (i) $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$

   (ii) $\begin{bmatrix} 3 & 2 & 5 \\ 6 & -5 & 3 \\ -24 & 38 & 2 \end{bmatrix}$ $\qquad$ 7 + 7

40085 $\qquad$ [Turn over

Group - II

3. a) A copper ball is heated to a temperature of 100°C. Then at time t = 0, it is placed in water which is maintained at 20°C. At the end of 3 minutes, the temperature of the ball is reduced to 60°C. The time rate of change of temperature T of the ball is proportional to the difference between T and the temperature of the surrounding medium. Assume that heat flows so rapidly in copper that at any time the temperature is practically the same at all points of the ball. Find the time at which the temperature of the ball is reduced to 25°C. 7

b) A cynderical tank 1.60 meter high stands on its circular base of diameter 1.2 meter and is initially filled with water. At the bottom of the tank there is a hole of diameter 1.0 cm, which is opened at some instant so that water starts draining under the difference of gravity. Find the height h (t) of water in the tank at any time, t. Find the time at which the tank is one–half full and completely empty. Given the velocity of water through the orifice is $v = 0.61\sqrt{2gh}$ when, g = 980 cm/sec². 7

4. Apply improved Enler — Canchy method to solve the following 'initial value problem' choosing h = 0.2 for the differential equation $y'x + y$, $y(0) = 0$. Find also the exact values for $X_n = 0$, 0.2, 0.4, 0.6, 0.6, 0.8 and 1.0 14
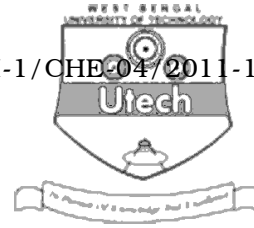
Group - III

5. a) Given $f(2) = 4$

   $F(2.5) = 5.5$

   Find the linear interpolating polynomial using (i) Lagrange interpolation, and (ii) Newton's divided difference interpolation.

   b) The following values of the function $f(x) = \sin x + \cos x$ are given

x	10°	20°	30°
$f(x)$	1.1585	1.2817	1.3660

   Construct the quadratic interpolating polynomial that fits the data. Hence find $f(\pi/12)$. Compare with exact value.   7 + 7

6. Construct the divided difference table for the following data :

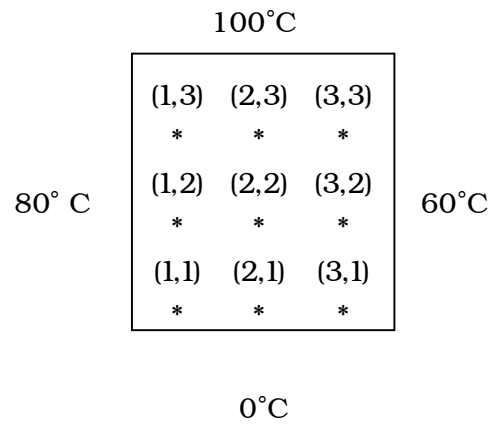 | x | 0.5 | 1.5 | 3.0 | 5.0 | 6.5 | 8.0 |
   |------|------|-------|-----|------|------|------|
   | f(x) | 16.2 | 58.75 | 310 | 1310 | 2821 | 5210 |

   Hence find the interpolating polynomial and an approximation to the value of $f(7)$.   14

Group - IV

7.   Use Liebman's method to solve the temperature of the heated plate as shown below, Employ overreloxation with a value of 1.5 for the weighing factor and iterate to $\xi_s = 1\%$

100˚C

(1,3)	(2,3)	(3,3)
*	*	*
(1,2)	(2,2)	(3,2)
*	*	*
(1,1)	(2,1)	(3,1)
*	*	*

80˚ C (left)   60˚C (right)

0˚C   14

8.   Solve the system of equations

$$\begin{pmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 1 \\ 1 & 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -10 \\ 8 \\ 7 \\ -5 \end{pmatrix}$$

using Gauss elimination method.   14