

Name :

Roll No. :

Invigilator's Signature :

CS/M.Tech (CHE)/SEM-1/CHE-04/2010-11

2010-11

**ADVANCE NUMERICAL METHODS IN CHEMICAL
ENGINEERING**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer any five questions. $5 \times 14 = 70$

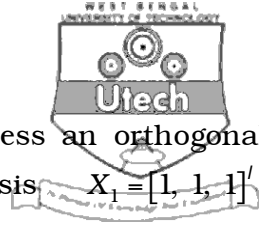
1. a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

- b) Find for what values of K , the following equations $x + y + z = 1$, $2x + y + 4z = K$ and $4x + y + 10z = K^2$ have solutions and solve them completely in each case.

2. a) Certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Find $\log_{10} 301$ using Lagrange's interpolation formula.

- b) Given $\frac{dy}{dx} = y - x$, where $x_0 = 0$, $y_0 = 2$ with $h = 0.2$. Determine y (0.2) correct to four decimal places by Runge-Kutta method. Find the analytical solution and find the error.



3. a) Construct using Gram-Schmidt process an orthogonal basis of $V_3(R)$, given a basis $X_1 = [1, 1, 1]^T$, $X_2 = [1, -2, 1]^T$, $X_3 = [1, 2, 3]^T$.
- b) Given the vectors $X_1 = [1, 2, 3]^T$ and $X_2 = [2, -3, 4]^T$, find
 - i) their inner product
 - ii) the length of each.
4. a) What is stiff-differential equation ? Solve the differential equation $\frac{dy}{dx} = -100y + 99e^{-x}$ with $y(0) = 0$ to justify the definition.
- b) Develop the Gear formula

$$\nabla y_{n+1} + \frac{1}{2}\nabla^2 y_{n+1} + \frac{1}{3}\nabla^3 y_{n+1} = hy'_{n+1} \quad \text{where } \nabla \text{ is the backward difference operator. Show that it is equivalent to } \nabla y_{n+1} = \frac{18}{11}y_n - \frac{9}{11}y_{n-1} + \frac{2}{11}y_{n-2} + \frac{6}{11}hy'_{n+1} \text{ and } y'_{n+1} = f(x_{n+1}, y_{n+1}).$$
5. Apply Galerkin's method to solve the boundary value problem (BVP) $y'' + y + x = 0$, $y(0) = y(1) = 0$.
6. Solve the BVP $y'' = -x^2$, $y(0) = y(1) = 0$ by finite element method.
7. Construct a set of polynomials $P_{m,N}(t)$ of degree $m = 0, 1, 2, \dots$ such that $P_{m,n}(t) = 0$ for $m > n$, so that the polynomials are orthogonal over the set of arguments t .



8. Determine the coefficients a_k so that

$$p(x) = a_0 P_{0,N}(t) + a_1 P_{1,N}(t) + \dots + a_m P_{m,N}(t) \text{ with}$$

$t = (x - x_0)/h$, will be the least squares polynomial of degree m for the data (x_i, y_i) $t = 0, 1, 2, \dots, N$.

9. Find the standard five-point formula of the finite difference analogue of Laplace's equation $u_{xx} + u_{yy} = 0$.
10. Use Gram-Schmidt orthogonalisation process to find the first two orthogonal polynomials on $[-1, 1]$ with respect to the weight function $w(x) = 1$.
