Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(BT)/INT.Ph.D (MICRO BIO/MOL BIO)/ SEM-1/(MBT/PHMC/PHMB)-104/2011-12

2011

INTRODUCTORY MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the Institute on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Answer any *ten* the following : $10 \times 1 = 10$

   i) Find the derivative of $f(x) = 3/\left(e^{5x}\right) + 4\,ln\left(1/\sqrt{x}\right) - (1/x)$.

   ii) Consider the functions : $f(t) = 2t^2 + t$ and $g(t) = t - 2$. Evaluate $f\left(g(1)\right)$.

   iii) The function $f(x) = \sin x + \cos x$ is

   a) even

   b) odd

   c) neither even nor odd.

iv) The equation $y = (2x+1)/(x-2)$ represents a ………………… .

v) Find A^{-1} if $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$.

vi) Find the value of x at which the function $f(x) = x^2 - 5x + 6$ has an extremum.

vii) Find the value of x for which the matrix $\begin{pmatrix} x & 4 \\ 3 & 2 \end{pmatrix}$ is singular.

viii) Write down the differential equation modelling the "pure death" process.

ix) Compute : $\int_0^{\pi/2} \log \tan x \, dx$.

x) Define the term "Wronskian".

xi) What is the domain of the function $f(x) = \sqrt{x^2 - 7x + 12}$?

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the equation : $\frac{dy}{dx}+2y=2,\ y(0)=0$.

3. If $u=\left(\frac{x}{y}\right)+\left(\frac{y}{z}\right)+\left(\frac{z}{x}\right)$, show that $x\left(\frac{\partial u}{\partial x}\right)+y\left(\frac{\partial u}{\partial y}\right)+z\left(\frac{\partial u}{\partial z}\right)=0$.

4. Find the eigenvalues and eigenvectors of the matrix $A=\begin{pmatrix}1 & 1\\ 4 & 1\end{pmatrix}$.

5. Compute : $\int \frac{\sin 2x}{a\sin^2 x+b\cos^2 x}dx$.

6. Find for what values of x, $f(x)=2x^3-21x^2+36x-20$ is maximum and minimum respectively. Find also the maximum and minimum values of the expression.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) When a person coughs, the trachea (windpipe) contracts, allowing air to be expelled at a maximum velocity. It can be shown that during a cough the velocity v of airflow is given by the function $v=f(r)=kr^2(R-r)$, where r is the radius of the trachea (in centimetres) during a cough, R is the normal radius of the trachea (in centimetres) and k is a positive constant that depends on the length of the trachea. Find the radius r for which the velocity of airflow is greatest. Draw a graph of the velocity of airflow v against the radius r.

b) Show that the frequency of heterozygotes in Hardy-Weinberg equilibrium, $H = 2p(1-p)$, has its maximum at $p = \frac{1}{2}$.

c) Given $\frac{x}{2} + \frac{y}{3} = 1$, find the maximum value of XY and the minimum value of $x^2 + y^2$. 7 + 3 + 5

8. a) The strength of a human body's reaction R to a dosage D of a certain drug is given by $R = D^2\left[(k/2)-(D/3)\right]$ where k is a positive constant. Show that the maximum reaction is achieved if the dosage is k units.

Also show that the rate of change in the reaction R with respect to the dosage D is maximal if $D = k/2$.

b) Using the method of Lagrange multipliers, find the relative minimum of the function $f(x, y) = 2x^2 + y^2$ subject to constraint $x + y = 1$.

c) Solve $\frac{dy}{dx} + y \tan x = \sin 2x$, $y(0) = 1$. 6 + 4 + 5

9. a) Solve the equation $x\left(\frac{dy}{dx}\right)+y^2=4$.

b) Solve the equation : $y'' - 4y = e^{-2x} - 2x,\ y(0)=0,\ y'(0)=0$.

c) The rate at which the concentration of a drug in the bloodstream decreases is proportional to the concentration at any time t. Initially, the concentration of the drug in the bloodstream is C_0 gm/mL. What is the concentration of the drug in the bloodstream at any time t ? Formulate (but do not solve) the problem in terms of a differential equation with an initial condition. 6 + 6 + 3

10. a) A mammalian cell line is susceptible to infection by virus like particles but can serve as host for only one such particle. The rate of infection equals half the number of uninfected cells, and the rate of elimination of the particles is half the number of infected particles. Set up a differential equation giving $\frac{dI}{dt}$ in terms of C and I, where I is the number of infected particles and C is the total number of cells. The number of cells is growing at a steady rate, so that $C = C_0 + 10t$, where C_0 is the initial number of cells. Solve the differential equation and show the eventually half the cells will be infected.

b) Under ideal laboratory conditions, the rate of growth of bacteria in a culture is proportional to the size of the culture at any time t. Suppose that 10,000 bacteria are present initially in a culture and 60,000 are present 2 hours later. How many bacteria will there be in the culture at the end of 4 hours ? 9 + 6

11. a) Find the eigenvalues of the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$.

Hence determine the eigenvectors corresponding to the smallest and the largest eigenvalue.

b) Solve the following differential equation by the method of variation of parameters :

$y'' + 2y' + y = e^{-x} \cos x$ 8 + 7