ENGINEERING & MANAGEMENT EXAMINATIONS, JANUARY – 2008

ADVANCED ENGINEERING MATHEMATICS

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Questions No. 1 and any *six* questions from the rest.

1. Answer any *five* questions from the following : $5 \times 2 = 10$

a) Examine whether the matrix $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ is an orthogonal matrix or not.

b) When are two vectors of an inner product space said to be orthogonal to each other ?

c) Prove the change of scale property of a Fourier transform.

d) What is a wavelet ?

e) Find the Laplace transform of $f(t) = e^{-t} \cos 3t$.

f) Give the mathematical expression for the convolution sum of two sequences of discrete time signals.

g) Give an example of the transition probability matrix of a random walk with reflecting barriers having 5 states.

h) Define autocorrelation and autocovariance in a stochastic process.

2. a) Define linear dependence and independence of vectors.

b) Examine whether the vectors (1, 2, 3), (2, 3, 1), (– 3, – 4, 1) are linearly independent.

c) Define linear span of a set of vectors. $2 + 6 + 2 = 10$

3. a) What do you mean by orthogonal diagonalisation ?

b) What do you mean algebraic multiplicity and geometric multiplicity of an eigenvalue of a matrix ?

c) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

$2 + 2 + 6 = 10$

4. a) Define 'norm of a vector' and 'unit vector' in an inner product space.

b) In R^3 let $\alpha = (a_1, a_2, a_3)$, $\beta = (b_1, b_2, b_3)$. Determine whether the mapping
$R^3 \times R^3 \to R$ defined by

$(\alpha, \beta) = a_1 b_1 + (a_2 + a_3)(b_2 + b_3)$ is a real inner product or not.

c) Examine whether $T : R^3 \to R^3$ defined by

$T(x, y, z) = (x + 2y + 3z, 3x + 2y + z, x + y + z)$, $(x, y, z) \in R^3$ is a linear transformation or not. 2 + 4 + 4 = 10

5. a) Find the Fourier cosine series of the function

$f(x) = x$ in $0 < x < L$.

b) Given $x^2 = \dfrac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \dfrac{\cos nx}{n^2}$, $-\pi < x < \pi$, prove that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

(Parseval's formula may be used). 6 + 4 = 10

6. Find the Fourier transform of $f(x) = e^{-|x|}$ and deduce that

$$\int_0^{\infty} \left(\frac{\cos xt}{1 + t^2} \right) dt = \frac{\pi e^{-|x|}}{2} .$$ 10

7. a) Prove the following property of Laplace transform :

$L[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n} F(s)$.

b) Using Laplace transform solve the following initial value problem :

$y'' - 2y' + 2y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$. 5 + 5 = 10

8. a) Apply convolution theorem to evaluate the following inverse Laplace transform :

$$L^{-1}\left\{ \frac{1}{s(s^2 + 4)} \right\} .$$

b) Find the Z-transform of the sequence $\{ a_n \}$, $n = 0, 1, 2, \ldots\ldots$ where $a_n = (n-1)^2$. 4 + 6 = 10

9\. a) Determine the convolution of the following two signals :

The impulse response of an LTI system given by $h(n) = \{ 1, \underset{\uparrow}{2}, 1, -1 \}$ and the

input signal given by $x(n) = (\underset{\uparrow}{1}, 2, 3, 1 \}$.

b) Use Z-transform to solve the following difference equation :

$$y_{n+2} + 3y_{n+1} + 2y_n = 0, \; y_0 = 2, \; y_1 = 4.$$ 5 + 5 = 10

10\. a) Define Strict Sense Stationary (*SSS*) and Wide Sense Stationary (*WSS*) stochastic process.

b) An FM station is broadcasting a tone, $x(t) = 100 \cos(10^8 t)$ to a large number of receivers. The amplitude and phase of the received wave-form are functions of distance between the transmitter and the receiver and so are RVs. The ensemble of received waveforms is a stochastic process $X(t)$ where $X(t) =$ $A \cos(10^8 t + \theta)$, A and θ being RVs. If A be constant and θ be uniformly distributed in $[0, 2\pi]$ then show that $\{ X(t) \}$ is WSS. 4 + 6 = 10

11\. a) The transition probability matrix of a Markov chain $\{ X_n \}$, $n = 0, 1, 2, 3, \ldots\ldots$ having three states 1, 2, 3 is

$$P = \begin{vmatrix} 0\cdot1 & 0\cdot5 & 0\cdot4 \\ 0\cdot6 & 0\cdot2 & 0\cdot2 \\ 0\cdot3 & 0\cdot4 & 0\cdot3 \end{vmatrix}$$

and the initial probability vector is $p^{(0)} = (0\cdot7, 0\cdot2, 0\cdot1)$.

Find (i) $P(X_2 = 3)$, (ii) $P(X_3 = 2, X_1 = 3, X_2 = 3, X_0 = 2)$.

b) A man tosses a fair coin untill three heads occur in a row. Let $X_n = k$ if at the n-th trail the last tail occurred at the $(n-k)$-th trial, *i.e.* X_n denotes the longest string of heads ending at the n-th trial. Show that the process in Markovian. Find the transition matrix. Also determine whether the chain is irreducible and aperiodic. 3 + 7 = 10

END