Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech (ME/PE)/SEM-8/ME-807/2010

2010

FINITE ELEMENT METHODS AND ITS APPLICATION

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternati es for the following : $10 \times 1 = 10$

i) Isoparametric element is one in which

a) geome ry of the element is described by a parameter

b) displacement of the element is described by another parameter

c) both geometry & displacement of element are described by same parameter

d) none of these.

ii) The circular cylinder with longer length will be teated for 2-D case as

a) plane stress

b) plane strain

c) either plane stress or plane strain

d) none of these.

iii) In finite difference method, the first order derivative has approximation in order of $(\Delta x)^2$ as

a) $\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x}$

b) $\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x}$

c) $\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i\quad 1,j}}{2\,\Delta x}$

d) none of th se

iv) Which one is no re ated to FEM ?

a) Crank Nicolson method

b) Variat onal method

c) Galerkin method

d) Banded symmetric metrix.

v) $U = a_0 + bx + cy$ is the deformation filled in case of

a) constant strain field

b) linearly varying strain field

c) parabolic variation of strain field

d) cubic variation of stain field.

vi) A differetial equation is given by

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\,\partial y} + C\frac{\partial^2\phi}{\partial y^2} = 0.$$

The condition for the equation to be hyperbolic one is

a) $B^2 - 4AC > 0$ b) $B^2 - 4AC = 0$

c) $B^2 - 4AC < 0$ d) none of these.

vii) $f(\xi) = \xi^2 + 2\xi + 1$. The value of $\int_{-1}^{1} f(\xi)\,d\xi$ by two point method is

a) 2·0 b) 1·667

c) 2·667 d) none of these.

viii) For quadrilateral element the dimension of element stiffness matrix is

a) $4 \infty 4$ b) $2 \infty 4$

c $8 \infty 8$ d) $4 \infty 8$.

ix) Eigenvalue problem is suitable for

a) steady fluid flow problems

b) mechanical vibration analysis

c) stress field problems

d) temperature field problems.

x) Von Mises stress is given by the equation

a) $\sigma_{VM} = \frac{1}{\sqrt{3}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$

b) $\sigma_{VM} = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$

c) $\sigma_{VM} = \frac{1}{2} (\sigma_1 - \sigma_3)$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. A light weight beam of length L is clamped at both ends and subjected to load $w(x)$ per unit length. If $R = EI$ is the flexural rigidity of the beam the defection ϕ is given by the solution of

$$\frac{d^2}{dx^2}\left[R(x) \frac{d^2\phi}{dx^2} \right] = w(x) \quad 0 \le x \le L$$

$$\phi = \frac{d\phi}{dx} = 0 \text{ a } x = 0 \text{ and at } x = L$$

Construct the functional for this specified problem.

3. Write the finite difference equation (using central differencing scheme) for the temperature flow field expressed by equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.$$

4. Write down the minimum Potential Energy Theorem. Considering linear shape function, prove the element stiffness matrix

$$[K^e] = \frac{E_e A_e}{L_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

for one-dimensional problem, E_e, A_e and L_e represent Young's muldulus, area and length of the element resprectively.

5.

Dia.

Consider the truss elements as shown in figure. The x, y co-ordinates of the two nodes are indicated in figure. If $q = [\,1\,5,\ 1\,0,\ 2{\cdot}1,\ 4{\cdot}3\,]^T \times 10^2$ in,

a) detemine the vector q^l

b) determine the K matrix.

6. How do you compare finite element method with finite difference method in the context of numerical simulation ? Explain finite-element discretization.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Consider the functional I for minimization given by

$$I = \int_0^L \frac{1}{2} K \left(\frac{dy}{dx} \right)^2 dx + \frac{1}{2} h (a_0 - 800)^2$$

with $y = 20$ at $x = 60$. Given $K = 20$, $h = 25$ and $L = 60$. Determine a_0, a_1 and a_2 using the polynomial approximation $y (x) = a_0 + a_1 x + a_2 x^2$ in Rayleigh-Ritz method. 9

b) If a displacement field is described by

$$u = (- x^2 + 2y^2 + 6xy) \infty 10^{-4}$$

$$v = (3x + 6y - y^2) \infty 10^{-4},$$

determine Σ_x, Σ_y, γ_{xy} at the point $x = 1$, $y = 0$. 6

8. a)

Dia.

$E = 200$ GPa

$f = 77$ kN/m^3

The bar has unit thickness. Find the deflection at the free end under its own weight using division of 1 element and 2 elements. Compare the results with exact solution. 12

b) Stress and strain vectors are related by the equation

$$\sigma = D\,\Sigma$$

Find D for plane stress problem. 3

9. a) Find the element stiffness matrix for four noded quadrilateral. All definitions of matrix should be clearly written and derived. 12

b) Write down the Helmholtz equation. For what type of problems is this equation used ? 3

10. A composite wall consists of three materials, as shown in the figure. The outer temperature is T_o = 20°C. Convective heat transfer takes place on the inner surface of the wall with T_α = 800°C and h = 25 w/m^2 °C. Determine he temperature distribution in the wall. Compare with analy ical results.

Dia.

11. a) Discuss different steps in finite element method. 6

b) Name a few commercial FEM packages. 2

c) Define isoparametric elements. 2

d) State the role of post-processor in FEM. 2

e) Write Euler-Lagrange equation for functional of one variable. 3