

CS/B.Tech/ME(NEW)/SEM-6/ME-605B/2013
2013
FINITE ELEMENT METHOD

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following : .

10 × 1 = 10 .

- i) FEA is a/an
 - a) analytical technique
 - b) computational technique
 - c) differential technique
 - d) none of these.
- ii) The field variables are the
 - a) independent variables
 - b) neither dependent nor independent variables
 - c) dependent variables
 - d) none of these.

- iii) The interpolation functions are called
 - a) shape functions
 - b) blending functions
 - c) both shape functions and blending functions
 - d) all of these.
- iv) The stiffness matrix gives the information of
 - a) geometric behaviour
 - b) material behaviour
 - c) both geometric and material behaviour
 - d) none of these.
- v) The Castigliano's theorem is applicable in FEM for
 - a) rotational displacement
 - b) linear displacement
 - c) both rotational and linear displacement
 - d) none of these.
- vi) MWR stands for
 - a) Mass weighted residuals
 - b) Method of weighted residuals
 - c) Moment of weighted residuals
 - d) none of these.

vii) In plane stress problems (considering X-Y planes), which entity is zero ?

- a) Strain along Z-direction
- b) Stress along Z-direction
- c) Stress along X-direction
- d) Stress along Y-direction.

viii) Discretization is based on

- a) Geometric dissimilarity
- b) Property dissimilarity
- c) Loading dissimilarity
- d) All of these.

ix) Natural co-ordinates are

- a) local co-ordinates
- b) euclidean co-ordinates
- c) global co-ordinates
- d) polar co-ordinates.

x) For better FEA, finite element size should be

- a) increased
- b) decreased
- c) unaltered
- d) infinitesimally small.

xi) Serendipity elements have nodes

- a) at the corners as well as at boundaries
- b) only at corners
- c) at the corners as well as at the interior of the element
- d) none of these.

xii) In 2-dimensional problems, degree of freedom for each node is

- a) one
- b) two
- c) three
- d) four.

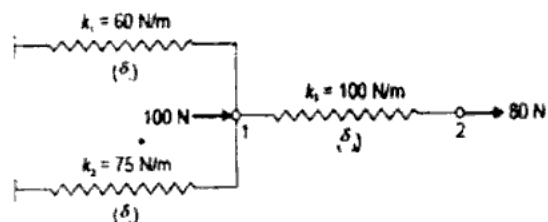
GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. What are the desirable features of finite element packages in computer implementations ?
3. Using generalized coordinate approach, find shape functions for two noded bar/truss element.
4. Find the strain displacement matrix of CST element.

5. Determine the displacements of nodes 1 and 2 in the spring system shown in the figure. Use minimum of potential energy principle to assemble equations of equilibrium.



6. Consider the functional I for minimization given by

$$I = \int_0^L \frac{1}{2} K \left(\frac{dy}{dx} \right)^2 dx + \frac{1}{2} h (a_0 - 800)^2 \text{ with}$$

$$y = 20 \text{ at } x = 60. \text{ Given } K = 20, h = 25, L = 60.$$

Determine a_0 , a_1 and a_2 using polynomial approximation

$$y(x) = a_0 + a_1 x + a_2 x^2 \text{ in the Rayleigh-Ritz method.}$$

GROUP - C

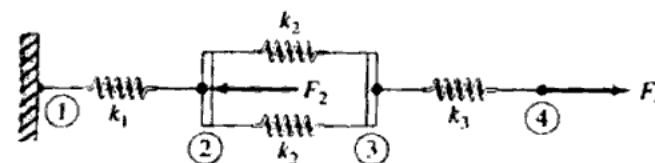
(Long Answer Type Questions)

Answer any three of the following. $3 \times 15 = 45$

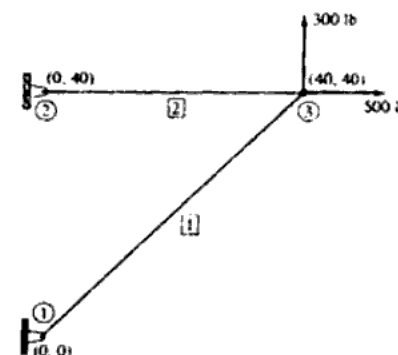
- a) Apply Castigliano's first theorem to the system of four spring elements depicted in the figure to obtain the system stiffness matrix. The vertical members at nodes 2 and 3 are to be considered rigid.

- b) Solve for the displacements and the reaction force at node 1 if $k_1 = 4 \text{ N/mm}$, $k_2 = 6 \text{ N/mm}$, $k_3 = 3 \text{ N/mm}$.

$$F_2 = -30 \text{ N}, F_3 = 0, F_4 = 50 \text{ N.} \quad 9 + 6$$



8. The two-element truss in the figure is subjected to external loading as shown. Using the same node and element numbering as in the figure, determine the displacement components of node 3, the reaction force components at nodes 1 and 2, and the element displacements, stresses and forces. The elements have modulus of elasticity $E_1 = E_2 = 10 \times 10^6 \text{ lb/in}^2$ and cross-sectional areas $A_1 = A_2 = 1.5 \text{ in}^2$.



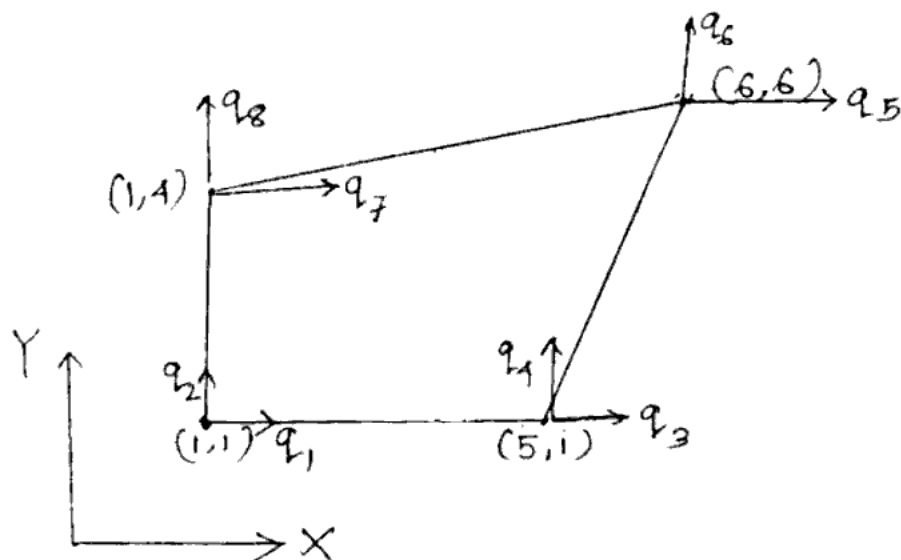
9. Use Galerkin's method to formulate a linear finite element for solving the differential equation

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} - 4x = 0 \quad 1 \leq x \leq 2$$

$$\text{subject to } y(1) = y(2) = 0.$$

10. The figure below shows a four node quadrilateral. The (x, y) co-ordinates of each node are given in the figure. The element displacement vector q is given by :

$$q = [0, 0, 0.20, 0, 0.15, 0.10, 0, 0.05]^T$$



- Find the x, y co-ordinate of a point P whose location in the master element is given by $\xi = 0.5, \eta = 0.5$.
- Find u, v displacement of point P .
- Using a 2×2 rule, evaluate the integral

$$\int_A (x^2 + xy^2) dx dy$$

by Gaussian quadrature, where A denotes the region shown by above figure.

$$4 + 4 + 7$$

- Differentiate between plane stress and plane strain problems. Also give examples for each problem.
- For an axisymmetric problem, derive D -matrix (matrix which transforms strain into stress).

12. Three bar elements of same cross-sectional area (200 mm²) are of lengths 400 mm, 800 mm and 1200 mm respectively. They are joined together to form a single bar of 2400 mm length. Young's Modulus E of the materials of all bars is 2×10^5 N/mm² and Element Stiffness Matrix is given by $\frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Find the Global Stiffness Matrix of the composite structure.