

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.TECH(OLD)/SEM-2/M-201/2011**

**2011**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) The rank of the matrix  $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -1 & 2 \\ 1 & 4 & 3 \end{bmatrix}$  is

a) 3

b) 5/2

c) 2

d) 1.

ii) In the Newton's forward interpolation formula the value of  $u = (x - x_0) / h$  lies between

a) 1 and 2

b) - 1 and 1

c) 0 and  $\infty$

d) 0 and 1.



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- iii) The eigenvalues of a matrix  $A$  are 2 and 4. Then the eigenvalues of  $A^{-1}$  are
- a) 2, 4    b) 4, 4
- c) 0.25, 0.5                                      d) 0.5, 0.25.
- iv) The value of  $a$  for which the vectors  $(1, 2, 1)$ ,  $(a, 1, 1)$  and  $(1, 1, 2)$  are linearly dependent is
- a) 2    b)  $\frac{2}{3}$
- c) -1    d)  $\frac{3}{2}$ .
- v) The value of the determinant  $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix}$  is
- a) 0     b)  $abc$
- c)  $-abc$     d)  $2abc$ .
- vi) If the system of equations  $4x + 2y - 5z = 0$ ,  $x + \lambda y + 2z = 0$ ,  $2x + y - z = 0$  has a non-zero solution, then  $\lambda$  is
- a)  $\frac{1}{3}$     b)  $\frac{1}{2}$
- c) 1     d) none of these.
- vii) The integrating factor of  $\frac{dy}{dx} + y = 1$  is
- a)  $e^x$     b)  $x$
- c)  $e^2$     d) 2.

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viii) The degree and order of the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = y^5 \text{ are}$$

- a) 1, 2    b) 3, 2  
c) 5, 2    d) 2, 1.

ix) The general solution of  $y = px + f(p)$ , where  $p = \frac{dy}{dx}$  is

- a)  $y = c^2 x + f(c)$                           b)  $y = cx + f(c^2)$   
c)  $y = cx + f(c)$                           d)  $y = cx^2 + f'(c)$ .

x) The value of the integral  $\int_0^{\infty} e^{5t} t^3 dt$  is

- a)  $\frac{1}{625}$     b)  $\frac{6}{625}$   
c)  $\frac{6}{25}$     d) none of these.

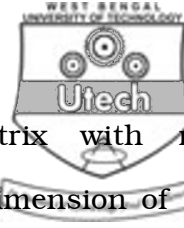
xi) Laplace transform of the function  $\cos at$  is

- a)  $\frac{s}{s^2 - a^2}$                                       b)  $\frac{s}{s^2 + a^2}$   
c)  $\frac{a}{s^2 + a^2}$                                       d)  $\frac{1}{s^2 - a^2}$ .

xii) The relation between  $E$  and  $\Delta$  is

- a)  $E \equiv 1 + \Delta$                               b)  $E \equiv 1 - \Delta$   
c)  $E \equiv \Delta - 1$                               d) none of these.

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xiii) Let  $A$  be an  $n \times n$  square matrix with row rank =  $r$  = column rank. Then the dimension of the space of solutions of the system of linear equations  $AX = 0$  is

- a)  $r$     b)  $n - r$   
 c)  $m - r$     d)  $\min(m, r) - r$ .

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following.                   $3 \times 5 = 15$

2. Solve the differential equation by Laplace transform  $\frac{d^2y(t)}{dt^2} + 4y(t) = \sin t, y(0) = \frac{dy(0)}{dx} = 0$ .

3. a) If  $S_1$  and  $S_2$  are two subspaces of a vector space  $V$ , then prove that  $S_1 \cap S_2$  is a subspace of  $V$ .

b) Show that  $S = \{ (x, y, z) \in \mathbb{R}^3 : x - 3y + 4z = 0 \}$  is a subspace of  $\mathbb{R}^3$ .

4. Show that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)$$

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5. Solve :

$$\begin{cases} \frac{dx}{dt} + y = e^t \\ \frac{dy}{dt} - x = e^{-t} \end{cases}$$

6. Evaluate :  $\int_0^2 (x^3 + 1) dx$  by Simpson's one-third rule

taking 4 intervals.

**GROUP - C****( Long Answer Type Questions )**Answer any *three* of the following.  $3 \times 15 = 45$ 7. a) Investigate for what values of  $\lambda$  and  $\mu$  the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

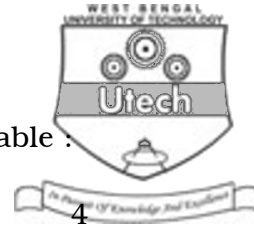
have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that

$$\begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

c) Show that every square matrix can be expressed as a sum of a symmetric and skew-symmetric matrix.

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8. a) Compute  $f(1 \cdot 3)$  from the following table :

$x :$	0	1	2	3	4
$f(x) :$	1	1.5	2.2	3.1	4.3.

b) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 2y = 3e^{-x} + x$$

c) If  $\alpha$ ,  $\beta$  and  $\gamma$  form a basis of a vector space  $V$ , then prove that  $\alpha + \beta + \gamma$ ,  $\beta + \gamma$  and  $\gamma$  also form a basis of  $V$ .

9. a) Show that the matrix

$$A = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \text{ is orthogonal and hence}$$

obtain  $A^{-1}$ .

b) Using the method of separation of symbols, prove that

$$u_0 + u_1 + u_2 + \dots + u_n = {}^{n+1}C_1 u_0 + {}^{n+1}C_2 \Delta u_0 +$$

$${}^{n+1}C_3 \Delta^2 u_0 + \dots + {}^{n+1}C_{n+1} \Delta^n u_0.$$

c) Show that

$$M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / \begin{matrix} a, b, c, d \in \mathbb{R} \\ a = d \end{matrix} \right\} \text{ is a subspace of the}$$

vector space of  $2 \times 2$  real matrices. Obtain a basis and dimension of  $M$ .

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10. a) If possible diagonalise the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix},$$

Specify the matrix which diagonalises  $A$  and the diagonal matrix to which  $A$  is changed after being diagonalised.

- b) Find  $L \left( \frac{1 - e^t}{t} \right)$ .
- c) Assuming orthogonal property of Legendre function, prove that

$$\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}.$$

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