

Name :

Roll No. :

Invigilator's Signature :

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2011

MATHEMATICS – II

Time Allotted : 3 Hours

Ful Marks : 70

The figures in the margin indicate full marks

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) The order and degree of the differential equation

$$\frac{d^2 y}{dx^2} = \left\{ y \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{1}{4}} \text{ is}$$

a) 2, 4

b) 4, 2

c) 1, 4

d) none of these.

ii) The integrating factor of the differential equation

$$\frac{dy}{dx} - 3y = \sin 2x \text{ is}$$

a) e^{3x}

b) e^{-3x}

c) e^x

d) none of these.

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iii) For the differential equation $f(x,y)\frac{dy}{dx} + g(x,y) = 0$ to be exact if

a) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

b) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

c) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 g}{\partial y^2}$

d) none of these.

iv) The auxiliary equation of $\frac{d^2 y}{dx^2} + a^2 y = \sin ax$ ($a \neq 0$) is

a) $m^2 + a^2 = 0$

b) $m^2 + 2a^2 = 0$

c) $m^2 + a = 0$

d) none of these.

v) The general solution of $y = px + \sqrt{a^2 p^2 + b^2}$, where $p = \frac{dy}{dx}$ is

a) $y = cx + \sqrt{a^2 c^2 + b^2}$

b) $y = cx - \sqrt{a^2 c^2 + b^2}$

c) $y = c - x\sqrt{a^2 c^2 + b^2}$

d) none of these.

vi) The maximum number of edges in a simple graph with n vertices is

a) n

b) $\frac{n-1}{2}$

c) $\frac{n(n-1)}{2}$

d) none of these.

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- vii) A binary tree has exactly
- a) two vertices of degree two
 - b) one vertex of degree 2
 - c) one vertex of degree one
 - d) none of these.
- viii) If a graph G has 7 vertices and 9 edges, then the size of the adjacency matrix is
- a) 7×7
 - b) 7×9
 - c) 9×9
 - d) none of these.
- ix) Dijkstra's algorithm is used to
- a) find maximum flow in a network
 - b) scan all vertices of a graph
 - c) find the shortest path from a specific vertex to another one
 - d) none of these
- x) The singularities of the integral $\int_{-1}^2 \frac{dx}{x(x-1)}$ are
- a) 0, 1
 - b) 1, 2
 - c) -1, 2
 - d) 0, 2.
- xi) The value of $\Gamma\left(\frac{1}{2}\right)$ is
- a) 2π
 - b) $\sqrt{\pi}$
 - c) $\frac{\pi}{2}$
 - d) none of these.

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xii) Laplace transform of the function $\sin at$ is

- a) $\frac{s}{s^2 + a^2}$ b) $\frac{s}{s^2 - a^2}$
 c) $\frac{a}{s^2 + a^2}$ d) $\frac{a}{s^2 - a^2}$.

xiii) The value of $\Gamma(m)\Gamma(1-m)$ is

- a) $\frac{2\pi}{\sin \pi}$ b) $\frac{3\pi}{\sin m\pi}$
 c) $\frac{\pi}{\sin m\pi}$ d) none of th se.

xiv) The value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ is

- a) $\sqrt{\pi}$ b) π
 c) $\frac{\pi}{2}$ d) none of these.

GROUP - B**(Short Answer Type Questions)**Answer any *three* of the following $3 \times 5 = 15$

2. Sol $e(D^2 - 5D + 6)y = e^x \cos x$ where $D = \frac{d}{dx}$.
3. Find the general and singular solution of $(y - px)(p - 1) = p$
 where $p = \frac{dy}{dx}$.

4. Evaluate $L^{-1}\left(\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right)$.

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5. Prove that a tree with n vertices has $(n - 1)$ edges.
6. Find the value of the improper integral $\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$.

GROUP - C**(Long Answer Type Questions)**Answer any *three* of the following. $3 \times 15 = 45$

7. a) Apply the variation of parameters to solve $\frac{d^2 y}{dx^2} + 4y = \sin 2x$.
- b) Solve : $(x^2 D^2 - xD + 4)y = x \ln(\log x)$ where $D = \frac{d}{dx}$.
- c) Show that $\int_{-\infty}^{\infty} x e^{-x^2} dx = 0$ 5 + 5 + 5
8. a) State convolution theorem. Using convolution theorem prove that $L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right) = \frac{t \sin at}{2a}$.
- b) Solve the following differential equation using Laplace transform $(D^2 + 6D + 9)y = 1$:
 $y(0) = 0, y'(0) = 1$ $\left[D = \frac{d}{dx} \right]$.
- c) Evaluate $\int_0^{\infty} e^{-3t} \sin t \cos t dt$ using Laplace transform. 6 + 5 + 4

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9. a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

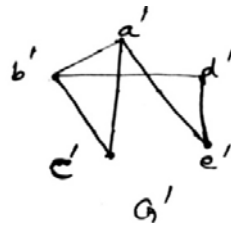
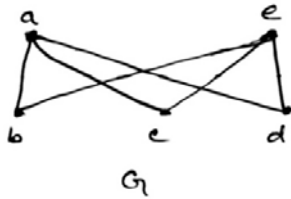
b) Evaluate $\int_0^1 x^2(1-x^2)^{\frac{7}{2}} dx$.

c) Prove that the number of odd degree vertices in a graph is always even. 5 + 5 + 5

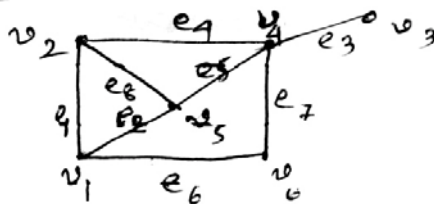
10. a) Draw the graph whose incidence matrix is

$$\begin{matrix}
 & e_1 & e_2 & e_3 & e_4 & e_5 \\
 v_1 & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix} \\
 v_2 & \\
 v_3 & \\
 v_4 &
 \end{matrix}$$

b) Define isomorphism. Examine whether the following two graphs are isomorphic or not.



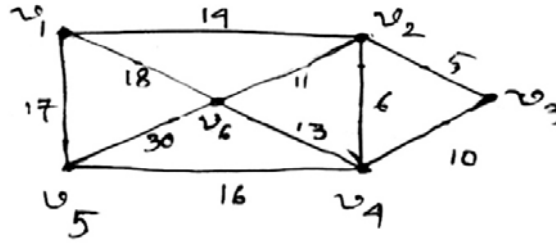
c) Determine the adjacency matrix of the given graph :



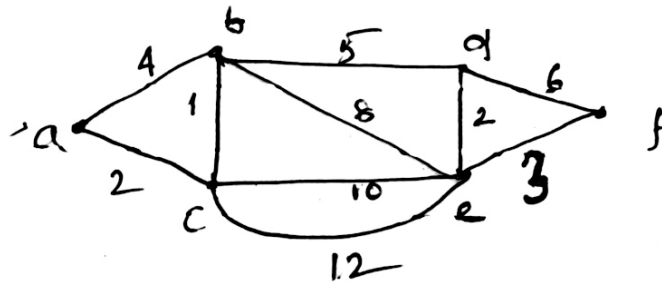
5 + 5 + 5

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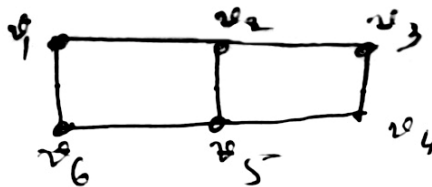
11. a) Apply Prim's algorithm to find the shortest spanning tree of the following graph and find the corresponding minimum weight.



- b) Find the shortest path from the vertex *a* to *f* in the following graph using Dijkstra's algorithm.



- c) Construct the spanning tree of the following graph by BFS algorithm.



$$6 + 5 + 4$$

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