

Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech(O)/SEM-1/M-101/2012-13

2012

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) $\lim_{n \rightarrow \frac{\pi}{2}} (1 - \sin x) \tan x =$

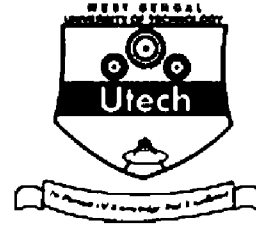
a) 1 b) 0

c) $\frac{1}{2}$ d) e .

ii) $\frac{dt}{dx}$ of $y = \sin^{-1} x + \sin^{-1} \sqrt{(1 - x^2)}$ is

a) 0 b) $\frac{1}{2}$

c) $\frac{x}{\sqrt{1 - x^2}}$ d) none of these.



iii) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ is

- a) convergent
- b) divergent
- c) neither convergent nor divergent
- d) none of these.

iv) If $u + v = x$, $uv = y$ then $\frac{\partial (x, y)}{\partial (u, v)} =$

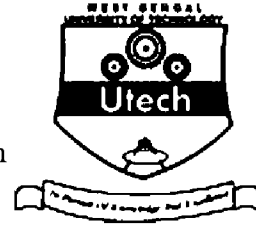
- a) $u - v$
- b) uv
- c) $u + v$
- d) $\frac{u}{v}$

v) If $u(x, y) = \tan^{-1} \left(\frac{y}{x} \right)$, then the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is

- a) 0
- b) $2u(x, y)$
- c) $u(x, y)$
- d) none of these.

vi) If $f(x)$ is continuous in $[a, a+h]$, derivable in $(a, a+h)$ then $f(a+h) - f(a) = hf(a + \theta h)$, where

- a) θ is any real
- b) $0 < \theta < 1$
- c) $\theta > 1$
- d) θ is an integer.



xi) If $f(x, y) = x^3 + 3xy^2 + y^3 + x^2$, then

$$x \frac{df}{dx} + y \frac{df}{dy} = 3f$$

- a) *True*
- b) *False.*

GROUP – B

(Short Answer Type Questions)

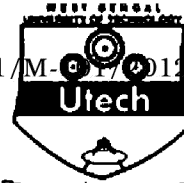
Answer any *three* of the following. $3 \times 5 = 15$

2. For what values of x the following series is convergent ?

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \dots$$

3. Find the moment of inertia of a thin uniform rectangular lamina of adjacent sides of lengths $2a$ and $2b$ and of mass M about an axis of symmetry through its centre.

4. Evaluate $\int_C (3xy \, dx - y^2 \, dy)$ where C is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.



5. If $y_n = \frac{d^n}{dx^n} \left\{ x^n \log_e x \right\}$, show that $y_n = 0$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

6. a) Verify Gauss divergence theorem for the vector field $\vec{F} = y \hat{i} + x \hat{j} + z^2 \hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0, z = 2$.
- b) Verify Stokes' theorem for $\vec{A} = (y - z + 2) \hat{i} + (yz + 4) \hat{j} - xz \hat{k}$ over the surface of the cube $x = y = z = 0$ and $x = y = z = 2$ above xy plane.

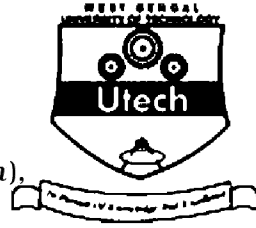
7 + 8

7. a) Using mean value theorem prove that

$$\frac{x}{1+x^2} < \tan^{-1} x < x, \quad 0 < x < \frac{\pi}{2}. \quad 5$$

- b) If z is a function of x and y and $x = r \cos \theta, y = r \sin \theta$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r} \frac{\partial z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 z}{\partial \theta^2}$.

5



c) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$,

$0 < \theta < 1, f(x) = 1 / (1 + x)$ and $h = 7$, find θ . 5

8. a) Show that for the function

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

$f_{xy}(0, 0) = f_{yx}(0, 0)$. 5

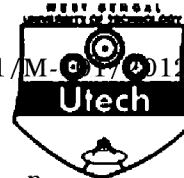
b) State comparison test for convergence of an infinite series. Test the convergence of any *one* of the following series :

(i) $\frac{6}{1.3.5} + \frac{8}{4.5.7} + \frac{10}{5.7.9} + \dots$

(ii) $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \dots (p > 0)$. 5

c) Find the extreme values, if any, of the following function :

$f(x, y) = x^3 + y^3 - 3axy$. 5



9. a) Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$. Hence

evaluate $\int_0^{\pi/2} \cos^5 x \, dx$. 5

b) Compute the value of $\iint_R y \, dx \, dy$ where R is the region

in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

c) Obtain the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$,

where m, n are positive integers ($m > 1, n > 1$). Hence
 evaluate $\int_0^{\pi/2} \sin^4 x \cos^8 x \, dx$. 5

