



Name :

Roll No. :

Invigilator's Signature :

**CS/B.Tech(CHE/FT)/SEM-3/M-315/2009-10
2009**

MATHEMATICS - III

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) If $\text{cov}(x, y) = 12$, $\sigma_x = 5$ and $r_{xy} = 0.6$, then the value of σ_y is

a) 16

b) 8

c) 2

d) 4.

ii) If the mean of a Poisson distribution is λ , then its standard deviation is

a) $\frac{1}{\sqrt{\lambda}}$

b) $\sqrt{\lambda}$

c) λ

d) $\frac{1}{\lambda}$.

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viii) The solution of the heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \text{ is}$$

- a) $u = (c_1 e^{px} + c_2 e^{-px}) e^{c^2 p^2 t}$
- b) $u = c_3 + c_4 x$
- c) $u = (c_5 \cos px + c_6 \sin px) e^{-c^2 p^2 t}$
- d) $u = (c_1 \cos px + c_2 \sin px)$.

ix) By elimination of a and b from $z = a(x + y) + b$, the partial differential equation formed is

- a) $\frac{\partial^2 z}{\partial x \partial y}$
- b) $\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} = 1$
- c) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$
- d) none of these.

x) The differential equation

$$\frac{\partial^2 u}{\partial x^2} + 2xy \sqrt{\frac{\partial z}{\partial x}} + \frac{\partial z}{\partial y} = 5 \text{ is}$$

- a) linear of order 2 and degree 1
- b) linear of order 2 and degree 2
- c) non-linear of order 2 and degree 1
- d) non-linear of order 2 and degree 2.

xi) The partial differential equation by eliminating the arbitrary constants from

$$z = ax + by + \sqrt{a^2 + b^2} \text{ is}$$

- a) $z = px + qy + \sqrt{p^2 + q^2}$
- b) $z = px - qy + \sqrt{p^2 + q^2}$
- c) $z = px + qy - \sqrt{p^2 + q^2}$
- d) $z = px - qy - \sqrt{p^2 + q^2}$.



- xii) The graph of the periodic function $f(x)$ defined by $f(x) = x, -a < x \leq a$ and $f(x + 2a) = f(x)$ for all x is
- square waveform
 - saw-toothed waveform
 - triangular waveform
 - half-wave rectifier.
- xiii) The smallest period of the function $\sin\left(\frac{2n\pi x}{k}\right)$ is
- $\frac{k}{2n\pi}$
 - $\frac{k}{n}$
 - $\frac{2\pi}{n}$
 - $2n\pi$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

- If a random variable X follows a Poisson distribution such that $P(X = 1) = P(X = 2)$, find
 - the mean of the distribution
 - $P(X = 4)$.
- Obtain the general solution of the following partial differential equation

$$p \tan x + q \tan y = \tan z \text{ where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$
- Define Bessel's function of the first kind of order n, J_n .
Show that $J_{-n}(x) = (-1)^n J_n(x)$.
- Show that every function can be expressed as a sum of an even function and an odd function.
- Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = \sin(x + y)$.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of 'x on y' and of 'y on x' respectively, then find the interval in which k lies.

- b) If X is normally distributed with mean 3 and standard deviation 2, find c such that $P (X > c) = 2P (X \leq c)$.

$$\text{Given } \int_{-\infty}^{.42} \phi (t) dt = \frac{1}{3} .$$

- c) A random variable X has the following probability distribution :

$X = x_i :$	0	1	2	3	4	5	6	7
$P (X = x_i) :$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

- i) Determine the constant k .
- ii) Evaluate $P (X < 6)$, $P (X \geq 6)$, $P (3 < X \leq 6)$
- iii) Obtain the distribution function. $4 + 4 + 7$
8. a) Find a Fourier series expansion of the function $f (x) = x - x^2$, $-\pi < x \leq \pi$.

Hence, find the value of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

- b) Find the period of the function $f (x) = 2 | \cos^2 x |$. Draw its graph. Find which type of waveform is this ?
- c) Write down the Dirichlet's condition of Fourier series.

$7 + 5 + 3$



9. a) Solve $x \frac{\partial z}{\partial x}$, $y \frac{\partial z}{\partial y} = z$. Find a solution representing a surface meeting the parabola $y^2 = 4x$, $x = 1$.

- b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given that

$$u(0, t) = u(l, t) = 0, u(x, 0) = f(x) \text{ and}$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, 0 < x < l. \quad 7 + 8$$

10. a) Show that $\int_{-1}^1 P_n(x) P_m(x) dx = \begin{cases} 0, & \text{if } n \neq m \\ \frac{2}{2n+1}, & n = m \end{cases}$

- b) Use the method of Frobenius to find solution of the differential equation

$$2x^2 \frac{d^2 y}{dx^2} + (2x^2 - x) \frac{dy}{dx} + y = 0. \quad 8 + 7$$

11. a) Solve the partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = x^2 y.$$

- b) Find the value of the constant k such that

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function. Construct the distribution function and compute $P\left(x > \frac{1}{2}\right)$.

- c) Expand $f(x) = |x|$ in Fourier series in the interval $-\pi \leq x \leq \pi$.

$$\text{Hence prove that } 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}. \quad 5 + 5 + 5$$



12. a) If the mean of the following distribution is 1.46 then find the missing frequencies :

$x :$	0	1	2	3	4	5	Total
<i>Frequency :</i>	46	–	–	25	10	5	200

- b) Eliminate the arbitrary functions from the following partial differential equation :

$$z = f_1 (x + ay) + f_2 (x - ay) .$$

OR

Show that

$$J_{\frac{1}{2}} (x) = \sqrt{\frac{2}{\pi x}} \cos x .$$

- c) Let (x, y) and (u, v) represent two sets of bivariate data such that $u = ax + b$ and $v = cy + d$ then

$$r_{uv} = \frac{ac}{|a||c|} r_{xy} ; \text{ where } a, b, c, d \text{ are constants.}$$

5 + 5 + 5

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