Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.TECH(NEW)/BME/ECE/EE/EIE/PWE/ICE/EEE/SEM-3/M-302/2011-12

2011

MATHEMATICS – III

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Short Answer Type Questions)

1. Answer any *ten* of the following : 10 × 2 = 20

a) Evaluate $\oint_C \frac{z}{z^2 - 1}\,dz$ where $C : |z| = 2$.

b) Find the singularities of $f(z) = \frac{1}{\sin z - \cos z}$.

c) Find the Fourier Sine Transform of

$f(x) = \frac{1}{x}$.

d) Find the period of the function $f(x) = 2\,|\cos^2 x|$.

e) Prove that every function can be represented as a sum of even and odd function.

f) Show that $X = 0$ is a regular singular point but $X = 2$ is not a regular singular point of the equation $x(x-2)^3 y'' + 3(x-2)^3 y' + 4y = 0$.

g) Find the general value of $\log(-i)$.

h) From the discrete distribution find k :

X :	0	1	2	3	4	5	6	7
$P(X)$:	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Determine whether the function is even or odd :

$$f(x) = \log\left(x + \sqrt{x^2+1}\right).$$

j) If $f(x) = x, \quad 0 < x < a,$

$$= 0, \quad x > a,$$

find the Fourier Sine Transform of $f(x)$.

k) If $f(x) = e^{-x^2/2}$, then show that $F(s) \approx \sqrt{2\pi}\ e^{-s^2/2}$,

where $F(s)$ is the Fourier transform of $f(x)$.

l) If $f(x) = e^{-x}$, $x \geq 0$, show that Fourier sine transform of $f(x)$ is $s F'(s)$, where $F(s) = \tan^{-1} s$.

m) A random variable X has the density function $f(x) = x$, $0 \leq x \leq 1$, $f(x) = \frac{1}{2}$, $1 < x \leq 2$. Find the mean of X.

n) Express $x^3 + x^2$ in terms of Legendre Polynomials :

$$P_0(x), P_1(x), P_2(x), P_3(x).$$

o) A random variable X has the following probability density function :

X :	0	1	2	3	4
$P(X = x) = f(x)$:	0	5k	3k	k	k

Determine the value of k.

GROUP – B

(Long Answer Type Questions)

Answer any *five* questions from the following :

$5 \times 10 = 50$

2. i) Let $f(x) = x$, $0 < x < 2$. Find the half range coine series.

Write Parseval's Identity coresponding to $f(x)$. Hence show that

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots + \frac{1}{n^4} = \frac{\pi^4}{90}.$$

ii) For a function defined by $f(z) = \sqrt{|xy|}$; show that the Cauchy-Riemann equations are satisfied at $(0, 0)$ but the function is not differentiable at that point.

5 + 5

3. i) Obtain series solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0 \text{ near } x = 0 \text{ such that } y(0) = 1,\ y'(0) = 2.$$

ii) Expand the function $f(x) = \frac{1}{(z-1)(z-2)}$ between the annular region of $|z| = 1$ and $|z| = 2$. 5 + 5

4. i) If $U(x, y) = 4xy - x^3 + 3xy^2$, verify that U is harmonic function and obtain its conjugate $V(x, y)$ so that $f = U + iV$ is an analytic function and also find $f(z)$, where $z = x + iy$, $x, y \in R$.

ii) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} 1 & \text{for } 0 < x \le \pi \\ 0 & \text{for } x > \pi \end{cases}$$

and hence evaluate the integral

$$\int_0^\infty \frac{1 - \cos p\pi}{p} \sin px \, dp.$$ 5 + 5

5. i) A random variable X has the following :

p.d.f. $f(x) = cx^2$, $0 \le x \le 1$

$= 0$, otherwise

Find (i) C

(ii) $P\left(0 \le x \le \frac{1}{2}\right)$.

ii) a) Show that :

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0, \; m \neq n.$$

b) Show that $J_{-n}(x) = (-1)^n \, . \, J_n(x)$, $n \in N$ and J_n is Bessel function of first kind. 5 + 5

6. i) Use the method of Frobenius to solve the following differential equation :

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0.$$

ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $(x > 0, \; t > 0)$ subject to the conditions

a) $u(0, t) = 0$

b) $u(x, 0) = 1, \; 0 < x < 1$

$= 0, \; x \geq 1$

c) $u(x, t)$ is bounded. 5 + 5

7. i) Three bags contain respectively 4W, 7B, 9R; 5W, 8B, 7R and 6W, 9B, 5R balls. One ball is chosen at random from each bag. What is the probability that the balls are (a) of same colour (b) 2W and 1R ?

ii) A random variable X has the following probability mass function :

x	0	1	2	3	4	5	6
$P(X = x)$	k	3k	5k	7k	9k	11k	13k

a) Find the value of k

b) Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 5)$

c) Obtain the distribution function $F(x)$

d) What is the smallest value of x for which $P(X \leq x) > 0{\cdot}5$? 5 + 5

8. i) The radius of a circle has distribution given by the p.d.f.

$$f(x) = 1, \; 1 < x < 2$$
$$= 0, \text{ otherwise}$$

Find the mean and variance of the area of the circle.

ii) If the weekly wage of 10,000 workers in a family follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are

a) between Rs. 66 and Rs. 72

b) less than Rs. 66

c) more than Rs. 72. 5 + 5

9. i) Show that

$f(z) = [x^3(1+i) - y^3(1-i)]/(x^2+y^2)$, $z \neq 0$, $f(0) = 0$ is continuous and Cauchy's Reiman's equation are satisfied at the origin yet $f'(0)$ does not exist.

ii) Find the first four terms of the Taylor's series expansion of $f(z) = (z+1)/(z-3)(z-4)$ about $z = 2$ and also find the region of convergence. 5 + 5