# Name : <br> Roll No. : <br>  <br> Invigilator's Signature : <br> $\qquad$ <br> CS / B.TECH (EE (N), EIE, EEE, PWE, BME, ICE, ECE) / SEM-3 / M-302 / 2010-11 2010-11 MATHEMATICS 

Time Allotted : 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## GROUP - A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following :

$$
10 \times 1=10
$$

i) If $F[f(x)]=F(s)$ represents the Fourier transform of the function $f(x)$, then $F[f(a x)]$ (' $a$ ' being a constant) equals
a) $\quad F(s / a)$
b) $\quad a F(s)$
c) $\quad(1 /|a|) F(s / a)$
d) $\quad\left(1 / a^{2}\right) F(a s)$.

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ii) A function $f(x), a<x<b$, can be expanded in a Fourier series

a) only if it is continuous everywhere
b) even if it is discontinuous at a finite number of points in $(a, b)$
c) even if it is unbounded in $(a, b)$
d) only if it is both continuous $\&$ bounded in $(a, b)$.
iii) Three unbiased coins are tossed simultaneously. This is repeated four times. Then the probability of getting at least one head each time is
a) $\quad(1 / 8)^{4}$
b) $(2 / 8)^{4}$
c) $\quad(7 / 8)^{4}$
d) $\quad(3 / 8)^{4}$.
iv) For a Poisson distribution $P(X)$ is $P(1)=P(2)$, then $P(0)$ is
a) $1 / e$
b) $1 / e^{2}$
c) $1 / e^{3}$
d) none of these.
v) A graph has 10 vertices and 15 edges. Its circuit rank is
a) 25
b) 12
c) 6
d) 5 .

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vi) A binary tree has 11 vertices. The minimum and maximum height of the tree is

a) $(4,5)$
b) $(3,5)$
c) $(3,10)$
d) $(4,10)$.
vii) If $f(x)$ is an odd function then $\mathcal{F}(f(x))$ is given by
a) $\quad F(s)=2 F_{s}(s)$
b) $\quad F(s)=2 i F_{s}(s)$
c) $\quad F(s)=0 \cdot 5 i F_{s}(s)$
d) $2 F(s)=i F_{s}(s)$,
where $\mathcal{F}$ denotes Fourier Transform.
viii) The order of pole $z=0$ of the function $\frac{\cos Z}{z^{3}}$ is
a) 2
b) 1
c) 3
d) 4 .
ix) If $X$ is normally distributed with zero mean and unit variable, then the expectation of $X^{2}$, is
a) 1
b) 0
c) 8
d) 2 .
x) The maximum and minimum values for correlation coefficient are
a) 1,0
b) 2,1
c) $0,-1$
d) $1,-1$.

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xi) If a simple graph has 15 edges then sum of the degrees of all the vertices is

a) 25
b) 24
c) 50
d) 30 .
xii) A closed walk in which no vertex (except is terminal vertices) appear more than once is called
a) path
b) Eulerian circuit
c) circuit
d) trail.
GROUP - B

## ( Short Answer Type Questions )

Answer any three of the following

$$
3 \times 5=15
$$

2. If $f(z)=\frac{x y^{2}(x+i y)}{x^{2}+y^{4}}, z \neq 0 \& \quad f(0)=0$, then prove that
$\frac{f(z)-f(0)}{z} \rightarrow 0$ as $z \rightarrow 0$ along any radius vector but not as $z \rightarrow 0$ in any manner.
3. If $f$ is analytic function then show that $\nabla^{2}|f(z)|^{2}=4 \frac{\partial(u, v)}{\partial(x, y)}$ where $f(z)=u+i v$ and $z=x+i y$.
4. Expand the following function in a Fourier series in $[-\pi, \pi]$

$$
f(x)=\left\{\begin{array}{cc}
-\frac{1}{2}(\pi+x) & \text { when }-\pi \leq x<0 \\
\frac{1}{2}(\pi-x) & \text { when } 0 \leq x<\pi
\end{array}\right.
$$

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5. Show that $f(x)$ given by

$$
f(x)=\left\{\begin{array}{l}
x \\
k-x\left\{\begin{array}{l}
\text { for } 0<x<1 \\
\text { for } 1<x<2 \\
\text { elsewhere }
\end{array}\right. \text { is a probability density }
\end{array}\right.
$$

function for a suitable value of k . Calculate the probability that the random variable lies between $1 / 2$ and $3 / 2$.
6. Define isomorphism of two graphs. Show whether the following graphs are isomorphic or not :


## GROUP - C

## ( Long Answer Type Questions )

Answer any three of the following. $\quad 3 \times 15=45$
7. a) Consider Heavyside unit function

$$
\begin{aligned}
h(1-|t|) & =0,|t|>1 \\
& =1,|t| \leq 1
\end{aligned}
$$

Prove that $F^{-1}(\sin s / s)=h(1-|x|)$ where $F^{-1}$ is the inverse Fourier transform i.e., $F^{-1}(F(s))=f(t)$.

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b) Using Parseval's identity of Fourier transform paove that $\int_{0}^{\infty}(1-\cos x / x)^{2} d x=\pi / 2$

c) Using Fourier transform solve the heat equation $\delta^{2} u / \delta x^{2}=\left(1 / c^{2}\right)(\delta u / \delta x),-\infty<x<\infty, t>0$ with boundary condition $u(x, t) \rightarrow 0, \delta u(x, t) / \delta x \rightarrow 0$ as $|x| \rightarrow \infty \&$ initial condition $u(x, 0)=e^{-x^{2} / 4 c^{2}},-\infty<x<\infty$ $3+4+8$
8. a) Using Dijkstra's algorithm find the length of the shortest path of the following graph :

b) Find by Prim's Algorithm a minimum spanning tree from the following graph :


$$
8+7
$$

9. a) Solve the differential equation :
$k \partial^{2} u / \partial x^{2}=\partial u / \partial t,-\infty<x<\infty, t>0$
with $u(x, t)=0$ as $x \rightarrow \pm \infty, \partial u / \partial t=0$ as $x \rightarrow \pm \infty$ and $u(x, 0)=f(x),-\infty<x<\infty$.

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b) Apply Dijkstra's algorithm to determine a shoptest path between $a$ to $z$ in the following graph.

10. a) The probability density function of a random variable $X$ is $f(x)=K(x-1)(2-x)$, for $1 \leq x \leq 2$.
$=0$, otherwise.

Determine -
(i) the value of the constant k and
(ii) $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$.
b) In a normal distribution, 31\% of the items are under 45 and $8 \%$ are above 64. Find the mean and standard deviation. [Given that $P(0<Z<1.405)=0.42$ and $P(-0 \cdot 496<Z<0)=0 \cdot 19]$
c) If the equations of two Regression lines obtained in a correlation analysis are $3 x+12 y-19=0$ and $9 x+3 y=46$. Determine which one is Regression equation of $y$ on $x$ and which one is the regression equation of $x$ on $y$. Find the means of $x$ on $y$ and correlation coefficient between $x$ and $y$. $4+5+6$

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11. a) If $f(x)=\left\{\begin{array}{cc}0 & -\pi \leq x \leq 0 \\ \sin x & 0 \leq x \leq \pi\end{array}\right.$, prove that


$$
f(x)=\frac{1}{\pi}+\frac{1}{2} \sin x-\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2 n x}{4 n^{2}-1}
$$

Hence show that

$$
\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots . .=\frac{1}{2} .
$$

b) Evaluate $\int_{C} \frac{4-3 z}{(z-1) z(z-3)} \mathrm{d} z$, where $C$ is the circle $|z|=\frac{5}{2}$.
c) Show that $u(x, y)=x^{3}-3 x y^{2}$ is harmonic in $C$ and find a function $v(x, y)$ such that $f(z)=u+i v$ is analytic.

