Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(ECE-OLD)/SEM-7/EC-704-C/2009-10

2009

ADVANCED MATHEMATICS FOR ELECTRONICS ENGINEERING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) Bessel's function of first kind of order n is given by

a) $\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^n \frac{1}{r!\sqrt{n+r+1}}$

b) $\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{2r} \frac{1}{r!\sqrt{n+r+1}}$

c) $\sum_{r=0}^{\infty} (-1)^r \left(\frac{2}{x}\right)^{n+2r} \frac{1}{r!\sqrt{n+r+1}}$

d) $\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r!\sqrt{n+r+1}}$.

ii) An example of linear partial differential equation is

a) $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = x - y$

b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 4x$

c) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = xy$

d) $\frac{\partial^2 p}{\partial x \partial y} + \left(\frac{\partial p}{\partial x}\right)^2 = 0.$

iii) A regular singular point of the equation

$$10x^2 \cdot \frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0 \text{ is}$$

a) $x = -2$ b) $x = 0$

c) $x = 1$ d) $x = -1.$

iv) The value of the integral $\oint_c \frac{e^z}{z-2}\, dz$

where $c : |z - 2| = 6$ is

a) πie b) πi

c) $2\pi i e^2$ d) none of these.

v) If $f(z) = \frac{z^2 + 1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

a) 4 b) 1

c) 2 d) 3.

vi) The differential equation of the form

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0 \text{ is called}$$

a) Clairaut's equation   b) Legendre's equation

c) Bessel's equation   d) None of these.

vii) The residue of $\frac{z+1}{z^2-2z}$ at the pole $z = 2$ is

a) 2   b) $\frac{3}{2}$

c) $\frac{1}{2}$ d) 1.

viii) The graph of the periodic function

$$f(x)=\begin{cases}-k, & 0-a\le x<0\\ k, & 0<x\le a\end{cases}$$

and $f(x+2a)=f(x)$ for all x is a

a) square waveform

b) half-wave rectifier

c) triangular waveform

d) saw-toothed waveform.

ix) Simple poles of $\frac{z^2+4}{z^3+2z^2+2z}$ are $0, -1 \pm i$. This is

a) True   b) False.

x) Residue of $f(z)=\frac{z^2+4}{z^3+2z^2+2z}$ at $z = 0$ is

a) 1   b) 3

c) 2   d) 1.

xi) If $g(x)$ is a periodic function of period T, then the correct statement is

a) $g(x+T) = f(x) + T$

b) $g(x+T) = f(x) - f(T)$

c) $g(x+T) = f(x) - T$

d) $g(x+T) = g(x)$.

xii) If $f(x)$ is an odd function, then for the Fourier expansion of $f(x)$, given by

$$\frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$ the correct statement is

a) $a_o = 0, a_n = 0$

b) $a_o \neq 0, a_n = 0$

c) $a_o = 0, a_n \neq 0$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Obtain a residue of $f(z) = \dfrac{z^2}{z^2 + a^2}$ at $z = ia$.

3. Find a bilinear transformation that maps

$z = 1$, $z = i$ and $z = -1$ into $w = i$, $w = 0$

and $w = -i$ respectively.

4. If $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$, determine

$v(x, y)$ so that $f(z) = u(x, y) + iv(x, y)$ is an analytical function.

5. Show that $u(x, y) = x - y^2$ and $v(x, y) = \dfrac{y}{x^2 + y^2}$ satisfy Laplace's equation but $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$ not an analytic function.

6. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ between annular region bounded by $\{ z : |z| = 1 \}$ and $\{ z : |z| = 2 \}$.

7. Evaluate $\displaystyle\int_C \frac{4z^2 - 4z + 1}{(z-2)(z^2+4)}\, dz$ where $c = \{ z : |z| = 1 \}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Obtain a Fourier series to represent $f(x) = x^2$ in $-\pi \le x \le \pi$; and hence obtain the relation

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots$$

b) Determine poles of $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$, and obtain residue of $f(z)$ at each pole. 10 + 5

9. a) Use the method of contour integration to show that

$$\int_0^{2\pi} \frac{d\theta}{1 + a^2 - 2a\cos\theta} = \frac{2\pi}{1 - a^2} ;\ 0 < a < 1.$$

b) Establish $J_n(x) = \dfrac{x}{2n} \left[J_{n-1}(x) + J_{n+1}(x) \right]$

10 + 5

10. a) What are different types of Samplings ?

Explain them with illustrations.

b) Find Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$ 10 + 5

11. a) Expand $f(z) = \dfrac{z-1}{z+1}$ as a Taylor's series about $z = 1$ and determine the region of convergence.

b) Prove that $\displaystyle\int_0^\infty \frac{dx}{x^2+1} = \frac{\pi}{2}$, by using Cauchy's residue theorem. 8 + 7

12. a) Solve the equation $\dfrac{\partial u}{\partial t} = 3 \dfrac{\partial^2 u}{\partial x^2}$, $x > 0$, $t > 0$ subject to the conditions $u(x, 0) = e^{-x}$, $x > 0$; $u(0, t) = 0$ and $u(x, t)$ is bounded.

b) Solve the two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} , \frac{\partial^2 u}{\partial y^2} = 0, \; 0 \leq x \leq \pi, \; 0 \leq y \leq \pi$$ subject to

the conditions $u(x, o) = x$, $u(x, \pi) = 0$ and

$$\frac{\partial u(0,y)}{\partial x} = \frac{\partial u(\pi,y)}{\partial x} = 0.$$ 8 + 7

13. a) Express $J_4(x)$ in terms of J_0 and J_1.

b) Prove that $(2n+1)\,x\,P_n = (n+1)\,P_{n+1} + n\,P_{n-1}$.

c) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. 4 + 5 + 6