



Name :

Roll No. :

Invigilator's Signature :

**CS/B.Tech(ECE-OLD)/SEM-7/EC-704-C/2009-10
2009**

**ADVANCED MATHEMATICS FOR ELECTRONICS
ENGINEERING**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

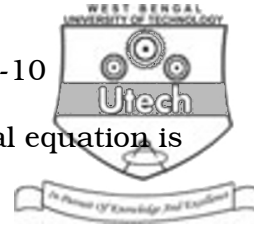
i) Bessel's function of first kind of order n is given by

a)
$$\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^n \frac{1}{r! \sqrt{n+r+1}}$$

b)
$$\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{2r} \frac{1}{r! \sqrt{n+r+1}}$$

c)
$$\sum_{r=0}^{\infty} (-1)^r \left(\frac{2}{x}\right)^{n+2r} \frac{1}{r! \sqrt{n+r+1}}$$

d)
$$\sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r! \sqrt{n+r+1}} .$$



ii) An example of linear partial differential equation is

a) $\sqrt{\frac{\partial z}{\partial x}} + \sqrt{\frac{\partial z}{\partial y}} = x - y$

b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 4x$

c) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = xy$

d) $\frac{\partial^2 p}{\partial x \partial y} + \left(\frac{\partial p}{\partial x}\right)^2 = 0.$

iii) A regular singular point of the equation

$$10x^2 \cdot \frac{d^2 y}{dx^2} + (2x^2 - x) \frac{dy}{dx} + y = 0 \text{ is}$$

a) $x = -2$

b) $x = 0$

c) $x = 1$

d) $x = -1.$

iv) The value of the integral $\oint_c \frac{e^z}{z-2} dz$

where $c : |z-2| = 6$ is

a) πe

b) πi

c) $2\pi i e^2$

d) none of these.

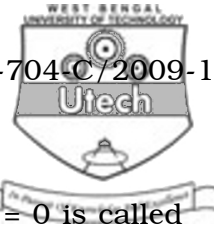
v) If $f(z) = \frac{z^2 + 1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

a) 4

b) 1

c) 2

d) 3.



vi) The differential equation of the form

$$(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0 \text{ is called}$$

- a) Clairaut's equation b) Legendre's equation
c) Bessel's equation d) None of these.

vii) The residue of $\frac{z+1}{z^2-2z}$ at the pole $z = 2$ is

- a) 2 b) $\frac{3}{2}$
c) $\frac{1}{2}$ d) 1.

viii) The graph of the periodic function

$$f(x) = \begin{cases} -k, & 0 - a \leq x < 0 \\ k, & 0 < x \leq a \end{cases}$$

and $f(x + 2a) = f(x)$ for all x is a

- a) square waveform
b) half-wave rectifier
c) triangular waveform
d) saw-toothed waveform.

ix) Simple poles of $\frac{z^2 + 4}{z^3 + 2z^2 + 2z}$ are $0, -1 \pm i$. This is

- a) True b) False.

x) Residue of $f(z) = \frac{z^2 + 4}{z^3 + 2z^2 + 2z}$ at $z = 0$ is

- a) 1 b) 3
c) 2 d) 1.



xi) If $g(x)$ is a periodic function of period T , then the correct statement is

- a) $g(x + T) = f(x) + T$
- b) $g(x + T) = f(x) - f(T)$
- c) $g(x + T) = f(x) - T$
- d) $g(x + T) = g(x)$.

xii) If $f(x)$ is an odd function, then for the Fourier expansion of $f(x)$, given by

$$\frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \text{the correct}$$

statement is

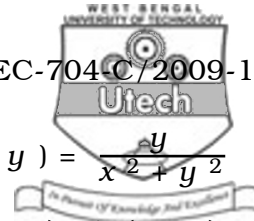
- a) $a_o = 0, a_n = 0$
- b) $a_o \neq 0, a_n = 0$
- c) $a_o = 0, a_n \neq 0$
- d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Obtain a residue of $f(z) = \frac{z^2}{z^2 + a^2}$ at $z = ia$.
3. Find a bilinear transformation that maps
 $z = 1, z = i$ and $z = -1$ into $w = i, w = 0$
 and $w = -i$ respectively.
4. If $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$, determine
 $v(x, y)$ so that $f(z) = u(x, y) + iv(x, y)$ is an analytical
 function.



5. Show that $u(x, y) = x - y^2$ and $v(x, y) = \frac{y}{x^2 + y^2}$ satisfy Laplace's equation but $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$ not an analytic function.
6. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ between annular region bounded by $\{z : |z| = 1\}$ and $\{z : |z| = 2\}$.
7. Evaluate $\int_C \frac{4z^2 - 4z + 1}{(z-2)(z^2+4)} dz$ where $c = \{z : |z| = 1\}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Obtain a Fourier series to represent $f(x) = x^2$ in $-\pi \leq x \leq \pi$; and hence obtain the relation
- $$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$
- b) Determine poles of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$, and obtain residue of $f(z)$ at each pole. 10 + 5
9. a) Use the method of contour integration to show that
- $$\int_0^{2\pi} \frac{d\theta}{1 + a^2 - 2a \cos \theta} = \frac{2\pi}{1 - a^2}; 0 < a < 1.$$
- b) Establish $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$

10 + 5



10. a) What are different types of Samplings ?

Explain them with illustrations.

b) Find Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases} \quad 10 + 5$$

11. a) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series about $z = 1$

and determine the region of convergence.

b) Prove that $\int_0^{\infty} \frac{dx}{x^2 + 1} = \frac{\pi}{2}$, by using Cauchy's

residue theorem.

8 + 7

12. a) Solve the equation $\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}$, $x > 0$, $t > 0$ subject

to the conditions $u(x, 0) = e^{-x}$, $x > 0$; $u(0, t) = 0$ and $u(x, t)$ is bounded.

b) Solve the two-dimensional Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq \pi, \quad 0 \leq y \leq \pi \text{ subject to}$$

the conditions $u(x, 0) = x$, $u(x, \pi) = 0$ and

$$\frac{\partial u(0, y)}{\partial x} = \frac{\partial u(\pi, y)}{\partial x} = 0. \quad 8 + 7$$



13. a) Express $J_4(x)$ in terms of J_0 and J_1 .
- b) Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$.
- c) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials.

4 + 5 + 6

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