WEST BENGAL UNIVERSITY OF TECHNOLOGY

CS-605A

OPERATION RESEARCH

Time Allotted: 3 Hours Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) An n dimensional convex polyhedron having exactly $(n+1)$ vertices is called

(A) Simplex (B) Convex hull
(C) Sphere (D) Triangle

(ii) For the following Linear Programming Problem -

Maximize $Z = 2x_1 - 3x_2$

S.t. $x_1 + x_2 \leq 2$

$2x_1 + 2x_2 \geq 8$

and $x_1, x_2 \geq 0$

$x_1 = 2.5, x_2 = 3.5$

(A) is a feasible solution but not a basic
(B) is a basic feasible solution
(C) is not a solution
(D) is a degenerate basic feasible solution

(iii) Find the range of values of p and q which will render the entry (2, 2) a saddle point for the game

strategies

	B_1	B_2	B_3
A_1	2	4	5
A_2	10	7	q
A_3	4	p	6

(A) $p \leq 7$ and $q \geq 7$
(B) $p = 7$ and $q = 7$
(C) $p \geq 7$ and $q \leq 7$
(D) $p \leq 7$ and $q \leq 7$

(iv) In EOQ inventory problem with no shortage in which demand is assumed to be fixed and completely pre-determined, the economic lot size is

(A) $\sqrt{\frac{2DC_o}{C_h}}$
(B) $\sqrt{\frac{2DC_h}{C_o}}$
(C) $\sqrt{2DC_oC_h}$
(D) none of these

where D is the demand rate, C_o is the ordering cost or set-up cost and C_h is the holding cost or carrying cost.

(v) A necessary and sufficient condition for a basic solution to a minimization type problem to be an optiomal is that (for all j)

(A) $Z_j - C_j \geq 0$
(B) $Z_j - C_j \leq 0$
(C) $Z_j - C_j = 0$,
(D) $Z_j - C_j < 0$ or $Z_j - C_j > 0$

(vi) A transportation problem is a balanced transportation problem iff -

(A) total demand and total supply are equal and number of sources equal to the number of destinations
(B) total demand equals to the total supply irrespective of the number of sources and destinations
(C) number of sources matches with the number of destinations
(D) the corresponding basic feasible solution is to be degenerate

(vii) In an assignment problem, the minimum number of lines covering all zeros in the reduced cost matrix of order n can be

(A) at most n (B) $n + 1$ (C) $n - 1$ (D) at least n

(viii) The steady state behavior of the queuing system which is in operation for a sufficient long period of time exists if

(A) $\lambda < \mu$ (B) $\lambda = \mu$ (C) $\lambda > \mu$ (D) $\lambda \leq \mu$

where λ is the mean arrival rate and μ is the mean departure rate

(ix) In (M/M/1):(N/FCFS) model, the steady state solution of the queuing system exists even for $\lambda \geq \mu$ because arrivals at the system are controlled by the system limit N not by the relative rates of arrival λ and departure μ

(A) True (B) False

(x) The time complexity of the Floyd's algorithm is

(A) $O(n)$ (B) $O(n^2)$ (C) $O(n^3)$ (D) $O(n^2 \log n)$

(xi) In a scheduling network,

(A) critical path means a longest path from start to end and there can be more than one such path

(B) critical path means a longest path from start to end and there can only one such path

(C) critical path means a shortest path from start to end and there can be more than one such path

(D) critical path means a shortest path from start to end and there can be only one such path

(xii) There are two products P and Q with the following characteristics:

Product	Demand(unit)	Order Cost(Rs/Order)	Holding Cost (Rs/unit/year)
P	100	50	4
Q	400	50	1

The Economic Order Quantity(EOQ) of products P and Q will be in the ratio

(A) 1:1 (B) 1:2 (C) 1:4 (D) 1:8

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. Find all the basic feasible solutions (if exist) of the equations

$$2x_1 + x_2 + 4x_3 = 11$$

and $$3x_1 + x_2 + 5x_3 = 14$$

Also, identify in each solution, the basic and non-basic variables.

3. Solve graphically the following LPP

Maximize $Z = 4x_1 + 3x_2$

Subject to the constraints

$$x_1 + x_2 \le 50,$$
$$x_1 + 2x_2 \le 80,$$
$$2x_1 + 2x_2 \le 20$$
$$x_1,\ x_2 \ge 0$$

4. Write the dual of the following problem

Minimize $Z = x_1 + x_2 + x_3$

Subject to

$$x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_2 \le 3$$
$$2x_2 - x_3 \ge 4$$

$x_1,\ x_2 \ge 0,\quad x_3$ is unrestricted in sign

5. For what values of λ, the game with the following payoff matrix is strickly determinable.

	B_1	B_2	B_3
A_1	λ	5	2
A_2	−1	λ	−8
A_3	−2	3	λ

6. Find the value of the game algebraically by using mixed strategies

Player B

Player A		B_1	B_2
	A_1	2	3
	A_2	4	−1

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Solve the LPP by simplex method 6+9

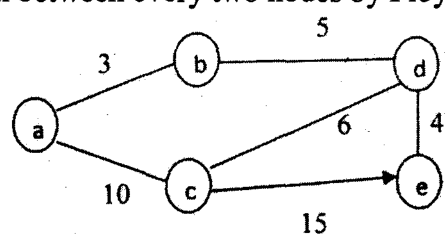
$$\text{Max } Z = 2x_2 + x_3$$

Subject to: $x_1 + x_2 - 2x_3 \leq 7$

$-3x_1 + x_2 + 2x_3 \leq 3$

$x_1, \quad x_2, \quad x_3 \geq 0$

(b) Find the shortest path between every two nodes by Floyd's algorithm.



Hence show the path from mode 'a' to node 'e' and find the shortest distance between them.

8. (a) The costs of transportation demand of warehouses and capacities of factories are given below in the following matrix. 9+6

	W_1	W_2	W_3	W_4	Capacities
F_1	2	1	3	4	30
F_2	3	2	1	4	50
F_3	5	2	3	8	20
Demands	20	40	30	10	

Find an optimal schedule of delivery for minimization of the cost of transportation.

Find an optimal solution of the transportation problem by VAM.

(b) Solve the assignment problem.

	M_1	M_2	M_3	M_4	M_5
J_1	160	130	175	190	200
J_2	135	120	130	160	175
J_3	140	110	155	170	185
J_4	50	50	80	80	110
J_5	55	35	70	80	105

9.(a) Prove that the probability of n customer in a (M/M/1): (∞/FIFO) Queue model is $P_n = \rho^n(1-\rho)$, where ρ is the traffic intensity. Also drive the expected queue length $L_Q = \frac{\rho^2}{1-\rho}$ and the expected length of system is $L_S = \frac{\rho}{1-\rho}$. 7+8

(b) Use dominance to reduce the pay-off matrix and solve the game with the following pay-off matrix:

		Player B B1	B2	B3
Player A	A1	3	-2	4
	A2	-1	4	2
	A3	2	2	6

10. Consider a project having the following activities and their time estimates. 3+7+5

Activity	predecessors	Time Estimates (Days)		
		t_o	t_m	t_p
A	–	2	4	6
B	A	8	12	16
C	A	14	16	30
D	B	4	10	16
E	C, B	6	12	18
F	E	6	8	22
G	D	18	18	30
H	F,G	8	14	32

(a) Draw a network diagram for this project.

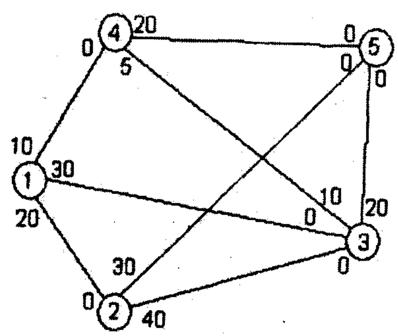
(b) Calculate the earliest and the latest expected times to each event and find the critical path.

(c) What is the probability that the project will be completed by 75 days?
[Given, $P(0 \le z \le 2.54) = 0.4945$]

11.(a) Solve the following Transportation Problem starting with the initial solution obtained by VAM. 7+8

	P	Q	R	S	Available
A	21	16	25	13	11
B	17	18	14	23	13
C	32	17	18	41	19
Requirement	6	10	12	15	43

(b) Find the maximum flow in the network.



 Turn Over