Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/B.TECH (CSE)/SEM-4/M(CS)-402/2011

2011
OPERATIONS RESEARCH & OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Graph sheet (s) will be supplied by the Institution.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The following system of equations

      $x_1 + x_2 + x_3 = 8$, $3x_1 + 2x_2 = 18$

      a) has no basic solution
      b) has (2, 6, 0), (6, 0, 2) as basic solutions
      c) has (1, 7, 0), (7, 1, 0) as basic solutions
      d) none of these

   ii) Any constraint in an LPP is expressed as.

      a) an equation with ' = ' sign
      b) inequality with ' $\geq$ ' sign
      c) inequality with ' $\leq$ ' sign
      d) any of these

iii) If there is no common region in the graphical method, then the LPP has no feasible solution.

a) True

b) False

iv) If the dual has an unbounded solution, then primal has

a) an unbounded solution

b) an infeasible solution

c) a feasible solution

d) none of these.

v) The solution of a transportation problem with m-source and n-destination is feasible if the number of allocation is

a) $m + n - 1$ b) $m + n + 1$

c) $m + n$ d) mn.

vi) In an assignment problem involving four workers and three jobs, the total number of possible assignments are

a) 4 b) 3

c) 7 d) 12

vii) Multiple servers may be

a) in parallel

b) in series

c) in combination of parallel and series

d) all of these.

viii) An activity is called critical activity if

a) $E_i = L_i$

b) $E_j = L_j$

c) $E_j - E_j = L_j - L_i = D_{ij}$

d) all these will satisfy together

ix) What is the method used to solve an LPP involving artificial variables ?

a) Simplex method

b) Charnes M – method

c) VAM

d) None of these.

x) If the maxmin and minmax values of a game are equal then

a) there is a saddle point

b) solution does not exist

c) strategies are mixed

d) none of these.

xi) In { (M/M/1) : (∞ / FIFO) }, average length of a non-empty queue is

a) $\frac{\lambda^2}{\mu(\mu-\lambda)}$ b) $\frac{\mu}{\mu-\lambda}$

c) $\frac{\lambda\mu}{(\mu-\lambda)^2}$ d) None of these.

Here the symbols have their usal meanings.

xii) Dijkstra's Algorithm is used to

a) find maximum flow in network

b) find the shortest path from a specified vertex to another

c) find the shortest path between any two vertices.

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$3 \times 5 = 15$

2. Define convex set. Show that $X = \{ x : |x| \leq 2 \}$ is a convex set.

3. solve graphically :

   Minimize $Z = x_1 + x_2$

   subject to $5x_1 + 9x_2 \leq 45$

   $x_1 + x_2 \geq 2$

   $x_2 \leq 4,\ x_1 \geq 0,\ x_2 \geq 0$

4. Find the dual of the following LPP :

   Maximize $Z = 2x_1 + 3x_2 - 4x_3$

   Subject to $3x_1 + x_2 + x_3 \leq 2$

   $-4x_1 + x_3 \geq 4$

   $x_1 - 5x_2 + x_3 = 5,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

5. In { (M/M/1) : (∞/FIFO) } queue, prove that average length of the queue $= \dfrac{\lambda}{\mu - \lambda}$,

   where λ = arrival rate and μ = service rate.

6. Solve the game whose pay-off matrix is given by

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{pmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{pmatrix} \end{array}$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following LPP by Simplex method :

Maximize $Z = x_1 - x_2$

subject to $x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$, $x_1 \geq 0$, $x_2 \geq 0$.

b) Solve the following LPP by Big-M method :

Maximize $Z = -2x_1 + x_2 + 3x_3$

subject to $x_1 - 2x_2 + 3x_3 = 2$

$3x_1 + 2x_2 + 4x_3 = 1$, $x_1, x_2, x_3 \geq 0$ 7 + 8

8. a) Solve the following assignment problem :

		Jobs 1	2	3	4	5
	A	6	5	8	11	16
	B	1	13	16	1	10
Man	C	16	11	8	8	8
	D	9	14	12	10	16
	E	10	13	11	8	16

b) Find the total cost of the following transportation problem by VAM method :

	D_1	D_2	D_3	
O_1	5	2	7	9
O_2	9	15	10	6
O_3	8	18	5	5
	5	5	10	

9 + 6

9. a) State the distinction between PERT and CPM.

b) A project consists of a series of tasks labelled A, B,....., H, I with the following relationships ($W < X$, Y means X and Y cannot start until W is completed; X, $Y < W$ means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints :

$A < D, E$; $B, D < F$; $C < G$; $B, G < H$; $F, G < I$.

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

5 + 10

10. a) Arrival rate of telephone calls of a telephone booth is according to a Poisson distribution with an average time of 9 minutes between two successive arrivals. The length of telephone call is assumed to be exponential distribution with mean 3 minutes.

(i) Find the time that a person arriving at the booth will have to wait.

(ii) Find average queue length.

(iii) The telephone company will instal a second booth when they have been convinced that an arrival would expected to have to wait at least 4 minutes for the phone.

Find the increase in flow of arrivals which will justify a second booth.

b) An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per price from a vendor. The requirement of these lubricants is 1800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise? 9 + 6

11. Using Dijkstra's algorithm find the shortest path and the length (or weight) of the shortest path of the following connected weighted graph from the vertex a to z.