*Name* : ..................................................................

*Roll No.* : ..................................................................

*Invigilator's Signature* : ..............................................

**CS/B.TECH(CHE)/SEM-8/CHE-804B/2012**

**2012**

# OPERATIONS RESEARCH

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Graph sheet(s) will be supplied by the Institute on demand.*

## GROUP – A
## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The system $x + y + z = 0$ has

a) a unique solution

b) no solution

c) infinite number of solutions

d) none of these.

ii) If we convert $x + y \geq 5$ into the equation $x + y + z = 5$, then $z$ is a

a) slack variable b) surplus variable

c) artificial variable d) none of these.

iii) Traffic intensity of a simple queue is given by

a) $\rho = \frac{\mu}{\lambda}$ b) $\rho = \frac{\mu t}{\lambda}$

c) $\rho = \frac{\mu}{\lambda t}$ d) $\rho = \frac{\lambda}{\mu}$.

iv) The variable (s) in the inventory is/are

a) controlled variable b) uncontrolled variable

c) both (a) and (b) d) none of these.

v) In Monte Carlo simulation

a) randomness is the key requirement

b) the model is of deterministic nature

c) both (a) and (b)

d) none of these.

vi) The dual of a dual problem is

a) primal b) dual

c) both (a) and (b) d) none of these.

vii) Solution of an assignment problem is

a) optimal & unique

b) always unique

c) may or may not be unique

d) does not exist.

viii) In a transportation problem with $m$ origin and $n$ destinations (balanced), the number of independent constants are

a) $m + n$ b) $mn$

c) $m + n - 1$ d) $m + n + 1$.

ix) In an assignment problem involving 5 workers and 4 jobs, total number of assignments possible is

a) 4 b) 5

c) 9 d) 20.

x) Operations Research is

a) applied decision theory

b) a scientific knowledge

c) a scientific approach to problem solving for executive management

d) all of these.

xi) Among the following which one is the better method ?

a) Matrix minima method

b) North-west corner rule.

xii) To find the shortest path we use

a) CPM

b) Floyd's algorithm.

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. Draw a network for the following project :

$A$ is the first and $K$ is the last operation of the project.

$F$ and $G$ can be done concurrently, but both must follow $A$.

$F$ must precede $H$.

$J$ cannot begin until both $F$ and $G$ are completed.

$K$ is dependent on the completion of both $H$ and $J$.

3. Make a graphical representation of the set of constraints in the following L.P.P. Find the extreme points of the feasible region. Find also the maximum value of the objective function.

   Maximize, $Z = x + y$

   subject to $x - y \geq 0$,

   $2x - y \leq -2$,

   $x + y \geq -10$

   and $x \geq 0$, $y \geq 0$

4. A manufacturer has two types of machines to choose. He must have at least three *'A'* type of machines and one *'B'* type of machine. The cost of the machine is Rs. 1,000 for the *'A'* type and Rs. 1,200 for the *'B'* type. The floor area taken up by the two types of machines are 4 $m^2$ and 5 $m^2$ respectively. The total cost must not exceed Rs. 15,000 and the total available floor area is 40 $m^2$. The manufacturer estimates the weekly profit from the output as Rs. 120 for each *'A'* type machine and Rs. 100 for each *'B'* type machine. Formulate the problem to an L.P.P. to maximize the profit of the manufacturer.

5. Find the dual of the following L.P.P. :

   Maximize, $Z = x_1 + 4x_2 + 3x_3$

   subject to $2x_1 + 3x_2 - 5x_3 \leq 2$

   $3x_1 - x_2 + 6x_3 \geq 1$

   $x_1 + x_2 + x_3 = 4$

   and $x_1, x_2 \geq 0$ ; $x_3$ is unrestricted in sign.

6. What is meant by inventory ? What are the main objectives of an inventory model ?

**GROUP – C**

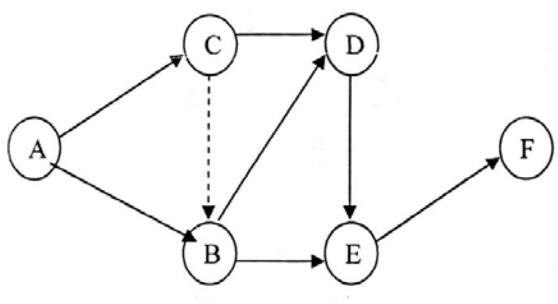
**( Long Answer Type Questions )**

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Define crashing of network and cost slope of an activity.

b) What are earliest expected time and latest allowable occurrence time ?

c) For the network shown in the figure, the three time estimates (in days) for each of the activities are indicated in the following table. Number of the events in the network according to the Fulkerson's rule in the steps of 10. If the schedule completion time is 30 days, determine the slack time for each event and identify the critical path. Enter the values in a tabular form.




| Activity | $t_O$ | $t_L$ | $t_P$ |
|---|---|---|---|
| A-B | 4 | 5 | 6 |
| A-C | 4 | 6 | 8 |
| B-D | 9 | 11 | 13 |
| B-E | 7 | 8 | 9 |
| C-B | 0 | 0 | 0 |
| C-D | 6 | 8 | 10 |
| D-E | 5 | 7 | 9 |
| E-F | 4 | 5 | 6 |

8. a) Solve the following L.P.P. by simplex method :

Maximize, $Z = x_1 + 2x_2$

subject to $x_1 - 5x_2 \le 10$

$2x_1 - x_2 \ge 2$

$x_1 + x_2 = 10$

$x_1, x_2 \ge 0$

Is the solution unique ? Justify your answer.

b) A salesman has to visit five cities $A$, $B$, $C$, $D$ and $E$. The distances (in hundred miles) between the five cities are as follows :

| | $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|---|
| $A$ | — | 7 | 6 | 8 | 4 |
| $B$ | 7 | — | 8 | 5 | 6 |
| $C$ | 6 | 8 | — | 9 | 7 |
| $D$ | 8 | 5 | 9 | — | 8 |
| $E$ | 4 | 6 | 7 | 8 | — |

If the salesman starts from city $A$ and has to come back at city $A$, which route should be selected so that the total distance travelled is minimum ?

9. a) A steel company has three open hearth furnaces and five rolling mills. Transportation costs (Rs. per quintal) for transporting steel from furnaces to rolling mills are shown in the following table :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $a_i$ |
|---|---|---|---|---|---|---|
| $F_1$ | 4 | 2 | 3 | 2 | 6 | 8 |
| $F_2$ | 5 | 4 | 5 | 2 | 1 | 12 |
| $F_3$ | 6 | 5 | 4 | 7 | 7 | 14 |
| $b_j$ | 4 | 4 | 6 | 8 | 8 | |

Find out the optimal transportation schedule using Vogel's approximation method. 10

b) Find the optimal assignment to find out the minimum cost for the assignment problem of the following cost matrix : 5

| | **Machine 1** | **Machine 2** | **Machine 3** |
|---|---|---|---|
| **Job 1** | 12 | 24 | 15 |
| **Job 2** | 23 | 18 | 24 |
| **Job 3** | 30 | 14 | 28 |

10. a) Define a queue. What are the basic characteristics of a queuing system ?

b) Prove that $L_q = L_s - \frac{\lambda}{\mu}$ of a $M/M/1$ queuing model.

c) In a supermarket, the average arrival rate of customer is 10 every 30 minutes following Poisson process. The average time taking by a cashier to list and calculate the customer's purchase is 2·5 minutes following exponential distribution. What is the probability that the queue length exceeds 6 ? What is the expected time spent by a customer in the system ? 5 + 5 + 5

11. a) Make a graphical representation of the set of constraints in the following L.P.P. Find the extreme points of the feasible region. Find also the maximum value of the objective function.

Maximize $Z = 6x_1 + 4x_2$

subject to $5x_1 + 7x_2 \leq 35$

$7x_1 + 5x_2 \leq 35$

$4x_1 + 3x_2 \geq 12$

$3x_1 + x_2 \geq 3$

and $x_1, x_2 \geq 0$

b) The annual demand of an item is 3200 units. The unit cost is Rs. 6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150, determine (i) EOQ, (ii) No. of order per year, (iii) The optimal cost. 5 + 10

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