



Name :

Roll No. :

Invigilator's Signature :

CS / B.TECH (CE) / SEM-3 / CE-301 / 2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

- i) The mean and standard deviation of a Standard Normal Distribution are respectively

- a) 1, 0 b) 0, 1
c) 0, 0 d) 1, 1.

- ii) The probability of getting 2 or 3 or 4 from a throw of single dice is

- a) $\frac{1}{6}$ b) $\frac{1}{2}$
c) 0 d) 1.

- 2



- xii) A box contains 6 white and 4 black balls. One ball is drawn. What is the probability is it that white ?

- a) $\frac{2}{5}$ b) $\frac{3}{5}$
c) $\frac{1}{5}$ d) $\frac{4}{5}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. A periodic function $f(x)$ with period 2π is defined as follows :

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Find the Fourier series at $x = \pi$.

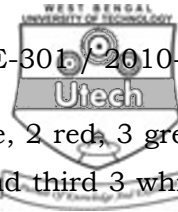
3. Solve the partial differential equation $z = px + qy + p^2 + pq + q^2$ and find its singular solution (The notations have their usual meanings).

4. Find the Fourier cosine transform of

$$f(x) = x, \quad 0 < x < 1$$

$$= 2 - x, \quad 1 < x < 2$$

$$= 0, \quad x > 2$$



5. There are three bags; first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.

6. Find the regression coefficients of y on x , of x on y and correlation coefficient between x and y from the following values :

$\sum xy = 1500$, $\bar{x} = 15$, $\bar{y} = 12$, $\sigma_x = 64$, $\sigma_y = 9$ and the number of observations is 10, where the notations have their usual meanings.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$$

- b) Using the method of separation of variable,

$$\text{solve } \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, u(0, t) = 0, u(4, t) = 0, u(x, 0) = \sin 3x.$$

6 + 9



8. a) Find the complete integral of the partial differential equation $p^2q(x^2 + y^2) = p^2 + q$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, $z = z(x, y)$.

- b) Find the Fourier series expansion of the periodic function of period 2π :

$$f(x) = x^2, -\pi \leq x \leq \pi$$

Hence, prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \infty = \frac{\pi^2}{12}$. 8 + 7

9. Solve the following heat condition equation :

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2} \text{ subject to the conditions}$$

$u(0, t) = 0, u(x, 0) = e^{-x}, x > 0, u(x, t)$ is bounded where $x > 0, t > 0$ using Fourier transform. 15

10. a) State Tchebycheff's inequality. Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$.

- b) State and prove Baye's theorem.

The three identical boxes I, II, III contain respectively 4 white and 3 red balls, 3 white and 7 red balls, and 2 white and 3 red balls. A box is chosen at random and a ball is drawn out of it. If the ball is found to be white, what is the probability the box II was selected ? 7 + 8



11. a) If X is a normal random variable $N(\mu, \sigma)$, then show that
 $E(X) = \mu$ and $Var(X) = \sigma^2$.

- b) Solve the following one dimensional wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2} \quad \text{with} \quad \left(\frac{\partial y}{\partial t} \right)_{t=0} = 0, y(x, 0) = f(x) \quad \text{using}$$

Fourier transform.

6 + 9

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