*Name* : ..............................................................

*Roll No.* : ..........................................................

*Invigilator's Signature :* .............................................

**CS / B.TECH (CE) / SEM-3 / CE-301 / 2010-11**

## 2010-11

# MATHEMATICS

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

### GROUP – A

### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The mean and standard deviation of a Standard Normal Distribution are respectively

a) 1, 0 b) 0, 1

c) 0 , 0 d) 1, 1.

ii) The probability of getting 2 or 3 or 4 from a throw of single dice is

a) $\frac{1}{6}$ b) $\frac{1}{2}$

c) 0 d) 1.

iii) The range of correlation coefficient is

a) $(-1, 1)$ b) $[-1, 1]$

c) $(-\infty, +\infty)$ d) none of these.

iv) When $f(x)$ convergence in the interval $(-\pi, \pi)$ then $\int_{-\pi}^{\pi} [f(x)]^2 dx$ is equal to

a) $\pi\left[\frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right]$ b) $2\pi\left[\frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right]$

c) $\left[\frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right]$ d) none of these.

v) If $F(s)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(ax)$, where $a(>0)$ is a constant, is

a) $F\left(\frac{s}{a}\right)$ b) $\frac{1}{a}F\left(\frac{s}{a}\right)$

c) $\frac{1}{a}F(s)$ d) $F(s)$.

vi) The partial deferential equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial t^2}, c^2$ being constant is known as

a) one dimensional wave equation

b) one dimensional heat-flow equation

c) two dimensional heat-flow equation

d) none of these.

vii) The integration value of $\int_0^\infty e^{-x^2} dx$ is

a) 0 b) $\pi$

c) $\sqrt{\pi}$ d) $\sqrt{\frac{\pi}{2}}$.

viii) $\cos(5x)$ is a periodic function with the period

a) $2\pi$ b) $\pi$

c) $\frac{2\pi}{5}$ d) none of these.

ix) The order and degree of the *p.d.e.* $\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial x}\right)^2 = 0$ are

a) 2, 2 b) 2, 1

c) 1, 2 d) none of these.

x) The equation $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ is

a) Parabolic b) Hyperbolic

c) Elliptic d) none of these.

xi) Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(AB) = \frac{1}{4}$. Then the value of $P(\overline{A}\,\overline{B})$ is

a) $\frac{5}{12}$ b) $\frac{1}{12}$

c) $\frac{7}{12}$ d) none of these.

xii) A box contains 6 white and 4 black balls. One ball is drawn. What is the probability is it that white ?

a) $2/5$
b) $3/5$
c) $1/5$
d) $4/5$.

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. A periodic function $f(x)$ with period $2\pi$ is defined as follows :

$$f(x) = \begin{cases} 0, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}$$

Find the Fourier series at $x = \pi$.

3. Solve the partial differential equation $z = px + qy + p^2 + pq + q^2$ and find its singular solution (The notations have their usual meanings).

4. Find the Fourier cosine transform of

$$f(x) = x, \; 0 < x < 1$$

$$= 2 - x, \; 1 < x < 2$$

$$= 0, x > 2$$

5. There are three bags; first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.

6. Find the regression coefficients of $y$ on $x$, of $x$ on $y$ and correlation coefficient between $x$ and $y$ from the following values :

$\sum xy = 1500$, $\bar{x} = 15$, $\bar{y} = 12$, $\sigma_x = 64$, $\sigma_y = 9$ and the number of observations is 10, where the notations have their usual meanings.

**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* of the following. 3 × 15 = 45

7. a) Solve the partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial x^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$$

b) Using the method of separation of variable,

solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = \sin 3x$ .

6 + 9

8. a) Find the complete integral of the partial differential equation $p^2 q (x^2 + y^2) = p^2 + q$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, $z = z(x, y)$.

b) Find the Fourier series expansion of the periodic function of period $2\pi$ :

$f(x) = x^2, \ -\pi \le x \le \pi$

Hence, prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots.\infty = \frac{\pi^2}{12}$. 8 + 7

9. Solve the following heat condition equation :

$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

$u(0, t) = 0, \ u(x, 0) = e^{-x}, \ x > 0, \ u(x, t)$ is bounded where $x > 0, \ t > 0$ using Fourier transform. 15

10. a) State Tchebycheff's inequality. Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least $\frac{19}{20}$.

b) State and prove Baye's theorem.

The three identical boxes I, II, III contain respectively 4 white and 3 red balls, 3 white and 7 red balls, and 2 white and 3 red balls. A box is chosen at random and a ball is drawn out of it. If the ball is found to be white, what is the probability the box II was selected ? 7 + 8

11. a) If $X$ is a normal random variable $N(\mu, \sigma)$, then show that $E(X) = \mu$ and $Var(X) = \sigma^2$.

b) Solve the following one dimensional wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 . \frac{\partial^2 y}{\partial x^2} \quad \text{with} \quad \left( \frac{\partial y}{\partial t} \right)_{t=0} = 0, y(x, 0) = f(x)$$ using Fourier transform. 6 + 9

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