	Utech
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# CS / B.TECH (CE) / SEM-3 / CE-301 / 2010-11 2010-11

### **MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

#### **GROUP - A**

## ( Multiple Choice Type Questions )

	$10 \times 1 = 10$
i)	The mean and standard deviation of a Standard Normal
	Distribution are respectively

Choose the correct alternatives for any ten of the following:

a) 1, 0

1.

b) 0, 1

c) 0,0

- d) 1, 1.
- ii) The probability of getting 2 or 3 or 4 from a throw of single dice is
  - a)  $\frac{1}{6}$

b)  $\frac{1}{2}$ 

c) 0

d) 1.

3153 [Turn over]



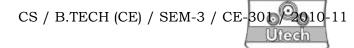
- The range of correlation coefficient is iii)
  - (-1, 1)

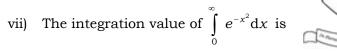
- [-1, 1] Annual (y)
- c)  $(-\infty, +\infty)$
- d) none of these.
- When f(x) convergence in the interval  $(-\pi,\pi)$ then  $\int_{0}^{\infty} [f(x)]^{2} dx$  is equal to
  - a)  $\pi \left[ \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right]$  b)  $2\pi \left[ \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right]$
  - c)  $\left[\frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)\right]$  d) none of these.
- v) If F(s) is the Fourier transform of f(x), then the Fourier transform of f(ax), where a(>0) is a constant, is
  - a)  $F\left(\frac{s}{a}\right)$

b)  $\frac{1}{a}F\left(\frac{s}{a}\right)$ 

c)  $\frac{1}{a}F(s)$ 

- The partial deferential equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial t^2}, c^2$  being vi) constant is known as
  - one dimensional wave equation a)
  - one dimensional heat-flow equation b)
  - two dimensional heat-flow equation c)
  - none of these. d)







c) 
$$\sqrt{\pi}$$

d) 
$$\sqrt{\frac{\pi}{2}}$$

viii)  $\cos(5x)$  is a periodic function with the period

a) 
$$2\pi$$

c) 
$$\frac{2\pi}{5}$$

d) none of these.

ix) The order and degree of the *p.d.e.*  $\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial x}\right)^2 = 0$  are

d) none of these.

x) The equation 
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$
 is

- a) Parabolic
- b) Hyperbolic

- c) Elliptic
- d) none of these.

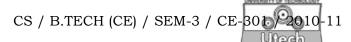
xi) Given  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{3}$ ,  $P(AB) = \frac{1}{4}$ . Then the value of  $P(\overline{A} \ \overline{B})$  is

a)  $\frac{5}{12}$ 

b)  $\frac{1}{12}$ 

c)  $\frac{7}{12}$ 

d) none of these.



- xii) A box contains 6 white and 4 black balls. One ball is drawn. What is the probability is it that white?
  - a)  $\frac{2}{5}$

b)  $\frac{3}{5}$ 

c)  $\frac{1}{\sqrt{5}}$ 

d)  $\frac{4}{5}$ 

#### GROUP - B

## (Short Answer Type Questions)

Answer any three of the following.

 $3 \times 5 = 15$ 

2. A periodic function f(x) with period  $2\pi$  is defined as follows:

$$f(x) = \begin{cases} 0, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}$$

Find the Fourier series at  $x = \pi$ .

- 3. Solve the partial differential equation  $z = px + qy + p^2 + pq + q^2$  and find its singular solution (The notations have their usual meanings).
- 4. Find the Fourier cosine transform of

$$f(x) = x, \ 0 < x < 1$$
$$= 2 - x, \ 1 < x < 2$$
$$= 0, x > 2$$

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- 5. There are three bags; first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.
- 6. Find the regression coefficients of y on x, of x on y and correlation coefficient between x and y from the following values:

 $\sum xy = 1500$ , x = 15, y = 12,  $\sigma_x = 64$ ,  $\sigma_y = 9$  and the number of observations is 10, where the notations have their usual meanings.

#### GROUP - C

#### (Long Answer Type Questions)

Answer any *three* of the following.  $3 \times 15 = 45$ 

7. a) Solve the partial differential equation

$$x^{2} \frac{\partial^{2} z}{\partial x^{2}} - 4xy \frac{\partial^{2} z}{\partial x \partial y} + 4y^{2} \frac{\partial^{2} z}{\partial x^{2}} + 6y \frac{\partial z}{\partial y} = x^{3} y^{4}$$

b) Using the method of separation of variable,

solve 
$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$
,  $u(0, t) = 0$ ,  $u(4, t) = 0$ ,  $u(x, 0) = \sin 3x$ .

6 + 9



- 8. a) Find the complete integral of the partial differential equation  $p^2q(x^2+y^2)=p^2+q$  where  $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y},$  z=z(x,y).
  - b) Find the Fourier series expansion of the periodic function of period  $2\pi$ :

$$f(x) = x^2, -\pi \le x \le \pi$$

Hence, prove that 
$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$
. 8 + 7

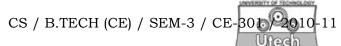
9. Solve the following heat condition equation:

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$$
 subject to the conditions

$$u(0,t) = 0$$
,  $u(x,0) = e^{-x}$ ,  $x > 0$ ,  $u(x,t)$  is bounded where  $x > 0$ ,  $t > 0$  using Fourier transform.

- 10. a) State Tchebycheff's inequality. Show by Tchebycheff's inequality that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least  $\frac{19}{20}$ .
  - b) State and prove Baye's theorem.

The three identical boxes I, II, III contain respectively 4 white and 3 red balls, 3 white and 7 red balls, and 2 white and 3 red balls. A box is chosen at random and a ball is drawn out of it. If the ball is found to be white, what is the probability the box II was selected? 7 + 8



- 11. a) If X is a normal random variable  $N(\mu, \sigma)$ , then show that  $E(X) = \mu$  and  $Var(X) = \sigma^2$ .
  - b) Solve the following one dimensional wave equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2} \quad \text{with} \quad \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0, y(x, 0) = f(x) \quad \text{using}$$

Fourier transform. 6 + 9

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