Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech (AUE)/SEM-4/AUE-401/2011

2011

ENGINEERING ANALYSIS & NUMERICAL METHODS

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : $10 \times 1 = 10$

   i) Which of the following is not true (the notations have their usual meaning) ?

   a) $\Delta = E - 1$ b) $\Delta.\nabla = \Delta - \nabla$

   c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\nabla = 1 - E^{-1}$.

   ii) If the interval of differencing is unity and $f(x) = ax^2$ (a is a constant), find which one of the following is wrong ?

   a) $\Delta f(x) = a(2x + 1)$ b) $\Delta^2 f(x) = 2a$

   c) $\Delta^3 f(x) = 2$ d) $\Delta^4 f(x) = 0$.

   iii) The value of $\Delta^{n+1} x^{(n)}$ is

   a) n ! b) 0

   c) n d) none of these.

iv) Newton's forward or backward interpolation formula is used for

a) equal intervals b) unequal intervals

c) both (a) and (b) d) none of these.

v) In the Simpson's $\frac{1}{3}$ rule, the no of equal sub-intervals is

a) even b) odd

c) both (a) and (b) d) none of these.

vi) The error in the composite trapezoidal rule is of order

a) h b) h^2

c) h^3 d) h^4.

vii) The area in Simpson's $\frac{1}{3}$ rule is approximated by the area of

a) rectangle b) parabola

c) ellipse d) none of these.

viii) The error in 4th order Runge-Kutta method is

a) $\sigma\left(h^2\right)$ b) $\sigma\left(h^3\right)$

c) $\sigma\left(h^4\right)$ d) $\sigma\left(h^5\right)$.

ix) In the method of interaction the function $\phi(x)$ must satisfy

a) $\left|\phi'(x)\right| < 1$ b) $\left|\phi'(x)\right| > 1$

c) $\left|\phi'(x)\right| = 1$ d) $\left|\phi'(x)\right| = 2$.

x) Gauss-elimination method fails when any one of the pivotal elements is

a) 0 b) 1

c) 2 d) – 1.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Show that $\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right) = \frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$ with $h = 1$.

3. Obtain the Newton-Raphson formula for finding the mth root of positive no. a and hence show that for the cube root of a (> 0) is $x_{n+1} = \frac{2x_n^3 + a}{3 \cdot x_n^2}$.

4. Solve by Gauss-elimination method

$$2x - y + 3z = 4$$
$$x + z = 2$$
$$2y + z = 3.$$

5. Solve $\frac{dy}{dx} = x + y$ with y (0) = 1, xt [0, 1] by Taylor's series method to obtain y for $x = 0{\cdot}1$.

6. Evaluate $\int_0^1 x.e^x dx$, using Trapezoidal rule taking $n = 6$.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Solve $x^2 + y^2 = 11$ and $y^2 + x = 7$, near $x = 3$ and $y = -2$ by Newton-Raphson method, correct to 3 decimal places.

   b) Find a positive root of $x^3 + x^2 - 1 = 0$ by the iterative method, correct to 4 decimal places. 8 + 7

8. a) Solve by Gauss-Jordan method $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$.

b) Find the inverse of the matrix by Gauss-Jordan method :

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

8 + 7

9. a) Find the largest eigenvalue and corresponding eigenvector of the matrix by power method.

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

b) Find a cubic polynomial which takes the following data, by Newton's forward interpolation. Hence find $f(0{\cdot}5)$:

x :	0	1	2	3
$f(x)$:	1	0	1	10

8 + 7

10. a) Compute $f^{I}(1{\cdot}1)$ and $f^{II}(1{\cdot}1)$ from the following table :

x :	1·1	1·2	1·3	1·4	1·5
$f(x)$:	2·0091	2·0333	2·0692	2·1143	2·1667

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$ rule, taking $h = \frac{1}{6}$. Hence calculate the value of λ. 8 + 7

11. a) Use 4th order Runge-Kutta method to find $y(0{\cdot}1)$ and $y(0{\cdot}2)$, correct to 4 decimal places when $\frac{dy}{dx} = y - x$, $y(0) = 2$.

b) Using Lagrange's polynomial find $y(10)$ if

x :	5	6	9	11
y :	12	13	14	16

8 + 7