



viii) The law of mean is given by

a) $\frac{f(b) + f(a)}{b - a} = f'(c)$

b) $\frac{f(b) + f(a)}{b + a} = f'(c)$

c) $\frac{f(b) - f(a)}{b - a} = f'(c)$

d) $\frac{f(b) - f(a)}{b - a} = f(c)$.

ix) If $x = r \cos \theta$ and $y = r \sin \theta$, then the value of $\frac{\partial (r, \theta)}{\partial (x, y)}$

is

a) 0

b) r

c) $\frac{1}{r}$

d) $-r$.

x) The series $\sum_{n=1}^{\infty} \frac{2}{e^n}$ is

a) convergent

b) divergent

c) oscillatory

d) none of these.

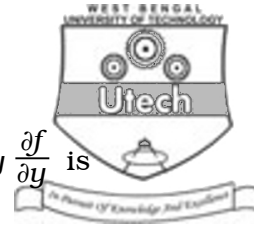
xi) The function $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ is

a) continuous and differentiable at $x = 0$

b) continuous but not differentiable at $x = 0$

c) neither continuous nor differentiable at $x = 0$

d) none of these.



xii) If $f(x, y) = \tan(x/y)$, then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is

- a) $\tan(x/y)$ b) $\cot(x/y)$
 c) 0 d) none of these.

xiii) The moment of inertia of a thin uniform rod of mass M and length $2a$ about an axis perpendicular to the rod at its centre is

- a) $\frac{Ma^2}{3}$ b) $\frac{Ma^2}{2}$
 c) Ma^2 d) $\frac{Ma^2}{4}$.

xiv) The point of intersection of the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-1}$ with the plane $x + 2y - z = 5$ is

- a) (1, 1, 1) b) (0, 1, 3)
 c) $(\frac{5}{3}, 1, \frac{-4}{3})$ d) none of these.

xv) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x \, dx$ is

- a) $I_n = \frac{n-1}{n} I_{n-1}$ b) $I_n = \frac{n}{n-1} I_{n-1}$
 c) $I_n = \frac{n-1}{n} I_{n-2}$ d) none of these.



GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. If $y = (x^2 - 1)^n$, then show that

$$(x^2 - 1) y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

3. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then show the

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2, \text{ where symbols}$$

have their usual meanings.

4. Test the convergence of the series

5. A, B, C and D are points $(\alpha, 3, -1), (3, 5, -3), (1, 2, 3)$ and $(3, 5, 7)$ respectively. If AB is perpendicular to CD, then find the value of α .

6. If _____, then prove that

7. Verify Rolle's theorem for the function

$$f(x) = |x|, -1 \leq x \leq 1.$$



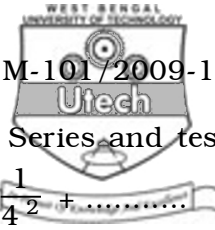
GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.

3 × 15 = 45

8. a) Examine continuity and differentiability of $f(x)$ at $x = 0$, when $f(x) = x \sin\left(\frac{1}{x}\right)$; ($x \neq 0$) and $f(0) = 0$.
- b) Show that
- is not continuous at $(0, 0)$
- c) Find the extrema of the function
- $f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$. 5 + 5 + 5
9. a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ and evaluate $\int_0^{\pi/2} \sin^5 x \, dx$.
- b) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$ then show that
- c) Prove that the function $f(x) = |x - 1|$, $0 < x < 2$, is continuous at $x = 1$, but not differentiable there. Is it continuous and derivable at $x = 0$? 5 + 5 + 5



10. a) State Leibnitz's theorem for Alternating Series and test convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

b) Define absolute and conditional convergence of Series. Also show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is absolutely

convergent.

6 + 9

11. a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction

$$\hat{i} - 3\hat{j} + 2\hat{k}.$$

b) Find the angles between the lines whose direction cosines are given by the equations $l + m + n = 0$ and

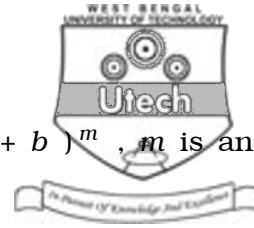
$$l^2 + m^2 - n^2 = 0.$$

c) Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

5 + 5 + 5

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12. a) Find the n -th derivative of $y = (ax + b)^m$, m is any number.

b) Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots$$

c) Find :

$\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$.

5 + 5 + 5

13. a) Find the whole length of the loop of the curve

$$9y^2 = (x - 2)(x - 5)^2.$$

b) Evaluate

c) State Green's Theorem.

6 + 6 + 3

