Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.Tech(O) / SEM-1 / M-101 / 2012-13

2012
MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) $\underset{n \to \frac{\pi}{2}}{Lt} \; (1 - \sin x) \tan x =$

a) 1 b) 0

c) $\frac{1}{2}$ d) e.

ii) $\frac{dt}{dx}$ of $y = \sin^{-1} x + \sin^{-1} \sqrt{(1 - x^2)}$ is

a) 0 b) $\frac{1}{2}$

c) $\frac{x}{\sqrt{1 - x^2}}$ d) none of these.

iii) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ is

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these.

iv) If $u + v = x,\ uv = y$ then $\frac{\partial(x,\ y)}{\partial(u,\ v)} =$

a) $u - v$ b) uv

c) $u + v$ d) $\frac{u}{v}$.

v) If $u(x,\ y) = \tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial}{\partial y}$ is

a) 0 b) $2u(x, y)$

c) $u(x, y)$ d) none of these.

vi) If $f(x)$ is continuous in $[a,\ a+h]$, derivable in $(a, a+h)$ then $f(a+h) - f(a) = hf(a + \theta h)$, where

a) θ is any real b) $0 < \theta < 1$

c) $\theta > 1$ d) θ is an integer.

vii) The value of $\int_{1}^{0} \int_{0}^{1} (x + y)\, dx dy = ?$

a) 2 b) 3

c) 1 d) 0.

viii) The series $\sum \frac{1}{n^p}$ is convergent if

a) $p \geq 1$ b) $p > 1$

c) $p < 1$ d) $p \leq 1$.

ix) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x\, dx$ is

a) $I_n = \left(\frac{n-1}{n} \right) I_{n-1}$

b) $I_n = \left(\frac{n}{n-1} \right) I_n$

c) $I_n = \left(\frac{n-1}{n} \right) I_{n-2}$

d) none of these.

x) The equation of the straight line passing through (1, 1, 1) and (2, 2, 2) is $(x - 1) = (y - 1) = (z - 2)$

a) *True*

b) *False.*

xi) If $f(x, y) = x^3 + 3xy^2 + y^3 + x^2$, then

$$x \frac{\mathrm{d}f}{\partial x} + y \frac{\mathrm{d}f}{\partial y} = 3f$$

a) *True*

b) *False*.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. For what values of x the following series is convergent ?

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \ldots.$$

3. Find the moment of inertia of a thin uniform rectangular lamina of adjacent sides of lengths $2a$ and $2b$ and of mass M about an axis of symmetry through its centre.

4. Evaluate $\int_C (3xy\,\mathrm{d}x - y^2\,\mathrm{d}y)$ where C is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

5. If $y_n = \frac{d^n}{dx^n}\left\{x^n \log_e x\right\}$, show that $y_n = ny_{n-1} + \lfloor n-1$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

6. a) Verify Gauss divergence theorem for the vector field $\bar{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$, $z = 2$.

b) Verify Stokes' theorem for $\vec{A} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the surface of the cube $x = y = z = 0$ and $x = y = z = 2$ above xy plane.

7 + 8

7. a) Using mean value theorem prove that

$$\frac{x}{1+x^2} < \tan^{-1} x < x, \quad 0 < x < \frac{\pi}{2}.$$ 5

b) If z is a function of x and y and $x = r\cos\theta$, $y = r\sin\theta$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2}$.

5

c) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$,

$0 < \theta < 1, f(x) = 1/(1+x)$ and $h = 7$, find θ. 5

8. a) Show that for the function

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

$f_{xy}(0, 0) = f_{yx}(0, 0)$. 5

b) State comparison test for convergence of an infinite series. Test the convergence of any *one* of the following series :

(i) $\frac{6}{1.3.5} + \frac{8}{4.5.7} + \frac{10}{5.7.9} + \ldots..$

(ii) $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \ldots\ldots. (p > 0)$. 5

c) Find the extreme values, if any, of the following function :

$f(x, y) = x^3 + y^3 - 3axy$. 5

9. a) Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$. Hence evaluate $\int_0^{\pi/2} \cos^5 x \, dx$. 5

b) Compute the value of $\iint_R y \, dx dy$ where R is the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

c) Obtain the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$, where m, n are positive integers ($m > 1, n > 1$). Hence evaluate $\int_0^{\pi/2} \sin^4 x \cos^8 x \, dx$. 5