

iii) If Rolle's theorem is applied to $f(x) = x(x^2 - 1)$ in $[0, 1]$, then $C =$

- a) 1
- b) 0
- c) $-\frac{1}{\sqrt{3}}$
- d) $\frac{1}{\sqrt{3}}$

iv) If $u + v = x$, $uv = y$, then $\frac{\partial(u, v)}{\partial(x, y)} =$

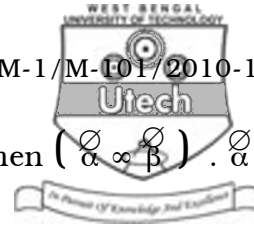
- a) $u - v$
- b) uv
- c) $u + v$
- d) $\frac{u}{v}$

v) The value of $\int_{-\pi/2}^{\pi/2} \sin^7 \theta \, d\theta$ is

- a) $\frac{6 \cdot 4 \cdot 2}{7 \cdot 5 \cdot 3 \cdot 1}$
- b) $\frac{6}{7}$
- c) 0
- d) $\frac{2 \cdot (6 \cdot 4 \cdot 2)}{7 \cdot 5 \cdot 3 \cdot 1}$

vi) The sequence $\left\{ (-1)^n \frac{1}{n} \right\}$ is

- a) convergent
- b) oscillatory
- c) divergent
- d) none of these.



vii) If $\vec{\alpha} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{\beta} = 2\hat{i} - \hat{k}$, then $(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\alpha}$ is equal to

- a) $\hat{i} + \hat{j} + \hat{k}$ b) $\hat{i} + \hat{k}$
 c) $\hat{i} - \hat{k}$ d) 0.

viii) The matrix $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is

- a) symmetric b) skew-symmetric
 c) singular d) orthogonal.

ix) The value of t for which

$\vec{f} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + tz)\hat{k}$ is solenoidal is

- a) 2 b) -2
 c) 0 d) 1.

x) The distance between two parallel planes $x + 2y - z = 4$ and $2x + 4y - 2z = 3$ is

- a) $\frac{5}{\sqrt{24}}$ b) $\frac{5}{24}$
 c) $\frac{11}{\sqrt{24}}$ d) none of these.



4. Show that

$$\begin{bmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{bmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right).$$

5. Test the nature of the series

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1 \cdot 2}{3 \cdot 5}\right)^2 + \left(\frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7}\right)^2 + \dots$$

6. If \vec{a} , \vec{b} , \vec{c} are three vectors, then show that

$$\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \right] = [a \ b \ c]^2.$$

7. If $u = \tan^{-1} \frac{x^2 - y^2}{x - y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$.

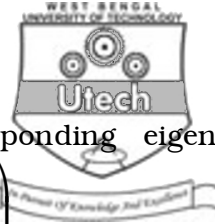
GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Determine the conditions under which the system of equations $x + y + z = 1$, $x + 2y - z = b$, $5x + 7y + az = b^2$, admits of

- i) only one solution
- ii) no solution
- iii) many solutions.



b) Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

c) Find whether the following series is convergent :

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots$$

9. a) If $f(x) = x^2$, $g(x) = x^3$ on $[1, 2]$, is Cauchy's mean value theorem applicable? If so, find ξ .

b) If $I_n = \int \frac{\cos n\theta}{\cos \theta} d\theta$, show that

$$(n-1)(I_n + I_{n-2}) = 2 \sin(n-1)\theta.$$

Hence evaluate $\int (4 \cos^2\theta - 3) d\theta$.

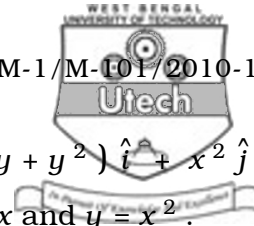
c) If $r = \left| \frac{\vec{r}}{r} \right|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that

$$\nabla (r^n) = nr^{n-2} \frac{\vec{r}}{r}.$$

10. a) Find $\frac{\partial}{\partial u}(u, v)$, where $u = x^2 - 2y^2$, $v = 2x^2 - y^2$

$\frac{\partial}{\partial (r, \theta)}$

and $x = r \cos \theta$, $y = r \sin \theta$.



- b) Verify Green's theorem for $\vec{F} = (xy + y^2)\hat{i} + x^2\hat{j}$ where the curve C is bounded by $y = x$ and $y = x^2$.

c) Evaluate :
$$\int_0^a \int_0^x \int_0^y x^3 y^2 z \, dz \, dy \, dx .$$

11. a) Find the maxima and minima of the function $x^3 + y^3 - 3x + 12y + 20$. Also find the saddle point.
- b) State Cayley- Hamilton theorem and verify the same for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Find A^{-1} and A^8 .
- c) Show that $\text{Curl } \nabla f = 0$,

where $f(x, y, z) = x^2 y + 2xy + z^2$.

12. a) Given the function
$$= \frac{xy(x^2 - y^2)}{x^2 + y^2}, (x, y) \neq (0, 0) \left. \vphantom{\frac{xy(x^2 - y^2)}{x^2 + y^2}} \right\}$$

$$= 0, (x, y) = (0, 0)$$

Find from definition $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$.

Is $f_{xy} = f_{yx}$?

- b) Integrate by Changing the order of integration

$$\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx.$$

- c) If $F(p, v, t) = 0$, show that

$$\left(\frac{dp}{dt}\right)_{v \text{ constant}} \times \left(\frac{dv}{dp}\right)_{t \text{ constant}} \times \left(\frac{dt}{dv}\right)_{p \text{ constant}} = -1.$$
