MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSM-101

PUID : 01004 (To be mentioned in the main answer script)

MATHEMATICS – 1A

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is

a) Symmetric b) Skew-symmetric

c) Singular d) Orthogonal.

ii) In the MVT $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$, if $f(x) = \frac{1}{1+x}$ and $h = 3$ then the value of θ is

a) 1 b) $\frac{1}{3}$

c) $\frac{1}{\sqrt{2}}$ d) none of these.

iii) The value of the integral $\int_{-\infty}^{\infty} x e^{-x^2} dx$ is

a) $2\sqrt{\pi}$

b) $-2\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) 0.

iv) If $T : V \to W$ is a linear transformation then Nullity of T + Rank of T equals to

a) dimension of V

b) dimension of W

c) dimension of $V + W$

d) none of these.

v) The sum of the eigenvalues of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 3 & 0 & -3 \end{pmatrix}$ is

a) 1

b) 5

c) 4

d) 0.

vi) If A is a skew-symmetric matrix then $P^T AP$ is

a) Symmetric

b) Skew-symmetric

c) Symmetric and Skew-symmetric

d) none of these.

vii) For which of the following function Rolle's theorem is *not* applicable ?

a) $f(x) = x^2 - 5x + 6$ in $[2, 3]$

b) $f(x) = \sin x$ in $[-\pi, \pi]$

c) $f(x) = \cos x$ in $[-\pi, \pi]$

d) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-\pi, \pi]$.

viii) If $\sin x = x - \frac{x^3}{\lambda} + \frac{x^5}{\mu} - \frac{x^7}{5040} + \dots$ then value of λ and μ are

a) 6 and 120 b) 60 and 1200

c) 600 and 1200 d) none of these.

ix) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) =$

a) $\frac{2\pi}{\sqrt{3}}$ b) $\frac{2\pi}{3}$

c) $\frac{\sqrt{3}\pi}{2}$ d) $\frac{3\pi}{2}$.

x) The value of '*a*' for which $\lim_{x \to 0} \frac{2x + a\sin 2x}{x^2}$ exists finitely is

a) 1 b) −1

c) 2 d) −2.

xi) If $\vec{F} = y^2 z\,\vec{i} + z^2 x\,\vec{j} + x^2 y\,\vec{k}$, then $\vec{F}$ is

a) Irrotational b) Solenoidal

c) both (a) and (b) d) none of these.

xii) The value of the paraboloid generated by revolving the part of the parabola $x^2 = 4ay$, $a > 0$ between the ordinates $y = 0$ and $y = a$ about its axis is

a) $2\pi a^3$ cubic units

b) $4\pi a^3$ cubic units

c) $\frac{4}{3}\pi a^3$ cubic units

d) $\frac{8}{3}\pi a^3$ cubic units.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Use Laplace's expansion to prove that

$$\begin{vmatrix} x & y & z & w \\ -y & x & w & -z \\ -z & -w & x & y \\ -w & z & -y & x \end{vmatrix} = (x^2 + y^2 + z^2 + w^2)^2 .$$

3. Using Lagrange's Mean Value Theorem prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$.

4. Solve the following system of equations by Gauss Jordan's method :

$2x - y - z = 0$, $x + 2y - z = 2$, $3x - y - z = 1$.

5. Prove that the vectors (1, 0, 1), (1, 1, 0), (1, –1, 1), (1, 2, –3) are linearly dependent.

6. Prove that the set $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$ is not a subspace of $\mathbb{R}^3$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$.

b) Prove that the determinant of every orthogonal matrix is 1 or –1.

c) Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

6 + 3 + 6

8. a) Prove that the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x - y, x + y, y)$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.

b) The line segment $x + y = 1$, $0 \le y \le 1$ is revolved about y axis to generate a curve. Find the lateral surface area of the cone.

c) Examine the function

$f(x, y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x$ for extrema

and indicate the saddle point, if any. 5 + 5 + 5

9. a) Diagonalize, if possible, the matrix $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

b) Use Cayley-Hamilton theorem to find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

c) Apply Gram-Schmidt process to the vectors (1, 0, 1), (1, 0, -1), (1, 3, 4) to obtain an orthogonal basis for $\mathbb{R}^3$ with the standard inner product. 6 + 4 + 5

10. a) Prove that $\int_0^{\pi/2} \sin^p x\,dx \times \int_0^{\pi/2} \sin^{p-1} x\,dx = \frac{\pi}{2(p+1)}$.

b) Using Mean Value Theorem prove that $0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1$.

c) Let T be a linear transformation of $\mathbb{R}^2$ into itself that maps (1, 1) to (–2, 3) and (1, –1) to (4, 5). Determine the matrix representation of T with respect to the basis { (1, 0), (0, 1) }. 5 + 5 + 5

11. a) Find the real value of z for which the rank of the matrix $\begin{pmatrix} 1+z & 2 & 3 & 4 \\ 1 & 2+z & 3 & 4 \\ 1 & 2 & 3+z & 4 \\ 1 & 2 & 3 & 4+z \end{pmatrix}$ is less than 4.

b) Find the basis and dimension of the Subspace W of $\mathbb{R}^3$ where $W = \{(x, y, z) : x + 2y + z = 0$ and $2x + y + 3z = 0\}$.

c) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, find the value of 'a' and the limit.

5 + 5 + 5