Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BNS/SEM-4/BNS-402/2010
2010
APPLIED MATHEMATICS – IV

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

    $10 \times 1 = 10$

    i) The value of $\begin{vmatrix} 2000 & 2001 & 2002 \\ 2003 & 2004 & 2005 \\ 2006 & 2007 & 2008 \end{vmatrix}$ is

    a) 0

    b) 100

    c) 1000

    d) none of these.

4083   [Turn over

ii)   If the determinant of a matrix $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 7 & 8 \\ 0 & 5 & -6 \end{pmatrix}$ is -26,

then the determinant of the matrix $\begin{pmatrix} 2 & 7 & 8 \\ 1 & 3 & 2 \\ 0 & 5 & -6 \end{pmatrix}$ is

   a)   26   b) -26

   c)   0   d)   84.

iii) If A be a nilpotent matrix such that $A^2 = 0$, then index of A is

   a)   1   b)   2

   c)   3   d)   4.

iv)   Lagrange interpolation formula is used for

   a)   equal interval

   b)   unequal interval

   c)   both equal and unequal intervals

   d)   none of these.

v)   The error in the Trapezoidal rule for the given interval $[\,a,\,b\,]$ is

   a) $-\dfrac{h^3}{12}\,f''(\,\xi\,)$ b) $-\dfrac{h^3}{12}\,f'(\,\xi\,)$

   c) $-\dfrac{h}{12}\,f''(\,\xi\,)$ d) $-\dfrac{h^2}{12}\,f''(\,\xi\,)$.

vi) Which of the following is not true (the notation have their usual meanings) ?

a) $\mu = \cosh \dfrac{hD}{2}$

b) $E^{1/2} = \mu + \delta$

c) $E = e^{hD}$

d) $\Delta = E - 1$.

vii) The median of the numbers 7, 2, 5, 9, 5 is

a) 2

b) 7

c) 9

d) 5.

viii) If cov $(x, y) = 10$, $\sigma_y = 5$ and $r_{xy} = 0\cdot4$, then $\sigma_x =$

a) 5

b) 25

c) 15

d) 125.

ix) Mean, median and mode are equal for

a) Binomial distribution

b) Normal distribution

c) Poisson distribution

d) Uniform distribution.

x) The probability of having at least one tail in 4 throws of a coin is

a) $\dfrac{7}{16}$

b) $\dfrac{9}{16}$

c) $\dfrac{11}{16}$

d) $\dfrac{15}{16}$.

xi) The value of t for which the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is singular, is

a) 3

b) 2

c) 1

d) $\dfrac{3}{2}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.   3 × 5 = 15

2. Prove that $(\Delta + \nabla)^2 (x^2 + x) = 8$, where $h = 1$.

3. Factorize $\Delta = \begin{vmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{vmatrix}$.

4. In a parking lot there are 12 places arranged in a row. Find the probability that a man coming for parking his car will find four consecutive places vacant.

5. Two dice are thrown simultaneously. Find the probability of obtaining (i) double six, (ii) no six.

6. If X follows normal distribution with mean 100 and variance 25, find the probability of

   $| X - 100 | \le 5$, *i.e.,* $P (| X - 100 | \le 5)$, given that

   $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1} e^{-x^2/2} \, dx = 0.8413447$.

GROUP – C

(Long Answer Type Questions)
Answer any *three* of the following.   3 × 15 = 45

7. a) Calculate $f(5)$ from the following data :

x	3	7	9	12
$f(x)$	11	17	24	30

5

b) Use Simpson's 1/3rd rule to estimate the value of $\log_e 2$ from $\displaystyle\int_0^3 \frac{\mathrm{d}x}{1+x}$, where the number of sub-intervals is 6.   5

c) Evaluate $\sqrt{28}$ to four decimal places by Newton's iterative method.   5

8. a) Find the equation of the line of regression of x on y for the following data :   5

x	1·0	1·5	2·0	2·5	3·0	3·5	4·0
y	5·3	5·7	6·3	7·2	8·2	8·7	8·4

b) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, then find the probability that (i) $X + Y = K$ where K is a constant, (ii) $X = Y$.   5

c) Calculate the median, quartiles and the quartile coefficient of skewness from the following data :

| Weight (lbs) :	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of persons :	12	18	35	42	50	45	20	8

5

9. a) Reduce the following matrix to an Echelon form and hence find its rank : 5

$$A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}.$$

b) If $\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ then verify that A satisfies its own characteristic equation. Hence find A^{-1}. 5

c) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$. 5

10. a) The demand for an item in a company is 18000 units per year and the company can produce the item at a rate of 3000 per month. The cost of one set-up is Rs. 500.00 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs. 20.00 per year. Determine the optimum manufacturing quantity and the number of shortages. Also, determine the manufacturing time and the time between set-ups. 2 + 2 + 2 + 1
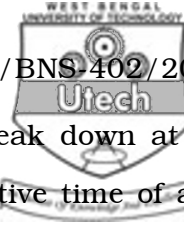
b) The machines in a production shop break down at an average of 2 per hour. The non-productive time of any machine costs Rs. 30 per hour. If the cost of repairman is Rs. 50 per hour and the repair rate is 3 per hour, calculate —

    i)   number of machines not working at any point of time.

    ii)   average time that a machine is waiting for the repairman.

    iii)   cost of non-productive time of the machine per hour.

    iv)   expected cost of the system per hour.   2 + 2 + 2 + 2

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