



Name :

Roll No. :

Invigilator's Signature :

CS/BNS/SEM-2/BNS-201/2011

2011

APPLIED MATHEMATICS – II

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

- i) The accuracy of Simpson's rule quadrature for a step size h is

- a) $O(h^2)$ b) $O(h^4)$
c) $O(h^3)$ d) $O(h^5)$.

- ii) The value of $\Gamma\left(\frac{7}{2}\right)$ is

- a) $\frac{15\sqrt{\pi}}{8}$ b) $\frac{15\sqrt{\pi}}{9}$
c) $\frac{5\sqrt{\pi}}{8}$ d) $\frac{25\sqrt{\pi}}{8}$.

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iii) The value of $\int_0^1 \int_0^1 \int_0^1 xyz dx dy dz$ is

a) $\frac{1}{6}$

b) $\frac{1}{8}$

c) $\frac{1}{27}$

d) $\frac{1}{16}$.

iv) The value of $erf(x) + erf_c(x)$ is

a) 0

b) 2

c) -1

d) 1.

v) The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots \infty$ is convergent if

a) $P \geq 1$

b) $P > 1$

c) $P < 1$

d) $P = 1$.

vi) The nature of the series $1 + 2 + 3 + 4 + \dots$ is

a) convergent

b) divergent

c) oscillatory

d) both convergent and divergent.



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vii) The Fourier series of $f(x) = x; 0 < x < 2\pi$ does not contain

- a) cosine term
- b) sine term
- c) constant term
- d) both sin and cosine terms.

viii) The value of the integral $\int_0^1 dx \int_0^1 e^{y/x} dx$ is

- a) $\frac{1}{2}(e+1)$
- b) $\frac{1}{2}(e-1)$
- c) $\frac{1}{2}(e+2)$
- d) $\frac{1}{2}(e+3)$.

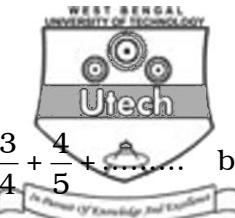
ix) The value of the integral $\int_0^{\frac{\pi}{2}} \sin^6 dx$ is

- a) $\frac{5\pi}{3}$
- b) $\frac{\pi}{32}$
- c) $-\frac{5\pi}{32}$
- d) $\frac{5\pi}{32}$.

x) The value of the error function $\text{erf}(0)$ is

- a) 1
- b) 2
- c) 3
- d) 0.

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xi) The nature of the series $1 + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots$ by

Cauchy's fundamental test is

- a) convergent
- b) divergent
- c) oscillatory
- d) both convergent and divergent.

xii) The value of $\int_0^{\infty} x^3 e^{-x} dx$ is

- a) 3
- b) 1
- c) 4
- d) 2.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$.
3. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
4. Prove that the moment of inertia of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of mass M about z -axis is $\frac{1}{3} M(a^2 + b^2)$, if the density at a point is proportional to xy .



5. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's one third rule taking

$$h = \frac{1}{6}.$$

6. Prove that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} (erf(b) - erf(a))$.

7. Evaluate $\iiint_R (x+y+z) dx dy dz$ where

$$R : 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Show that $\iint_R (x^2 + y^2) dx dy$ over the region R enclosed

by the triangle having its vertices at (0, 0), (1, 0) and

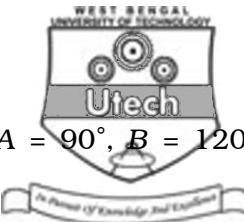
$$(1, 1) \text{ is } \frac{1}{3}.$$

b) Test the convergence of the following series :

i) $\frac{1}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{3}{5 \cdot 7} + \dots \dots \dots \infty$

ii) $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \dots \dots \dots \infty.$ $7 + 8$

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9. a) ABC is a spherical triangle in which $A = 90^\circ$, $B = 120^\circ$

$C = 60^\circ$. Find a , b , c .

b) In a spherical triangle ABC , angle $C = 90^\circ$, angle $B = 30^\circ$

and side $AB = 70^\circ$. Find side AC and the angle A .

c) In a spherical triangle ABC , angle $A = 88^\circ 24 \cdot 5'$,

$a = 87^\circ 01'$, $c = 100^\circ 09'$, $b = 98^\circ 10'$ find B . $5 + 5 + 5$

10. a) Find the C.G. of the area asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying on

first quadrant.

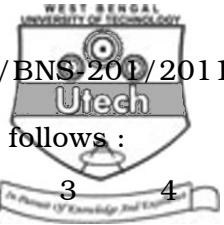
b) Find the Fourier Transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$.

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx = 1$. $7 + 8$

11. a) A ship 120 metres long at the waterline has equidistantly spaced half ordinates commencing from forward as follows :

$0, 3 \cdot 7, 5 \cdot 9, 7 \cdot 6, 7 \cdot 5, 4 \cdot 6, 0 \cdot 1$ meters respectively.

Find the area of the water-plane and the TPC at this draft.



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- b) The areas of a ship's water-planes are as follows :

Draft (m)	0	1	2	3	4
Area of WP (sq.m)	650	660	662	661	660

Calculate the ship's displacement in tonnes when floating in salt water at 4 meters draft. Also if the ship's load draft is 4 metres, find the FWA.

- c) A ship is floating upright on an even keel at 6·0 m draft

F and *A*. The areas of the water planes are as follows :

Draft (m)	0	1	2	3	4	5	6
Area (sq.m)	5000	5600	6020	6025	6025	6025	6025

Find the ship's *KB* at this draft.

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