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Paper Code : BMN-101

BASIC MATHEMATICAL COMPUTATION

Time Allotted: 3 Hours | *Full Marks*: 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternative for *any ten* of the following: 1×10=10

(i) The number of permutations that can be made out of the letters "COTTON" is

(a) 720 (b) 180

(c) 120 (d) 30

(ii) The value of $\int \frac{\log x}{x^2} dx$ is

(a) $\log(x+1)+c$ (b) $-\frac{1}{x}\log(x+1)+c$

(c) $\log(x-1)+c$ (d) $\frac{1}{2}\log(x+1)+c$

(iii) The function $f(x)=|x|$ is

(a) continuous and differentiable at $x=0$

(b) continuous everywhere but differentiable at $x=0$

(c) discontinuous and not differentiable at $x=0$

(d) None of the above

(iv) $f(x,y)=|x|+|y|$ then $f_x(0,0)$ equal to

(a) 1 (b) 0

(c) does not exist (d) None of these

(v) The value of y_3, when $y = \left(\frac{x}{2}+1\right)^8$ is

(a) $42\left(\frac{x}{2}+1\right)^3$ (b) $336\left(\frac{x}{2}+1\right)^3$

(c) $42\left(\frac{x}{2}+1\right)^5$ (d) $336\left(\frac{x}{2}+1\right)^5$

(vi) The equation of the straight line passing through the point (4,3) and making intercepts on the coordinate axes whose sum is −1

(a) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$ (b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(c) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$ (d) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

(vii) If $y = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$, then the value of $\frac{dy}{dx}$ at $x=-1$ is

(a) 1 (b) 0

(c) e (d) $1/e$

(viii) If A be a matrix whose inverse exists then which of the following is not true?

(a) $(A^T)^{-1} = \left(A^{-1}\right)^T$ (b) $(A^2)^{-1} = \left(A^{-1}\right)^2$

(c) $A^{-1} = (detA)^{-1}$ (d) None of these

(ix) If $y = 2at$ and $x = at^2$ then $\frac{dy}{dx}$ at $t=2$ is

(a) 1 (b) 2

(c) $2a^2$ (d) 1/2

(x) Which of the following does not satisfy Rolle's theorem in [−2,2]?

(a) x (b) $\frac{1}{x^2}$

(c) $\frac{1}{x-5^2}$ (d) x^2-5

(xi) The value of $\lim_{x\to 2}[x]$ is

(a) 2 (b) 1

(c) 3 (d) Does not exist

(xii) The angle between the lines $y=x-3$ and $y=\left(2-\sqrt{3}\right)x$ is

(a) 30° (b) 45°

(c) 60° (d) 90°

Group – B

(Short Answer Type Questions)

Answer *any three* of the following. 5×3=15

2. How many license plates can be formed involving 3 English letters and 4 digits, if the letters must appear either in the beginning or in the end?

3. If $y = \cos\left(m \sin^{-1} x\right)$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$.

4. If $u = \tan^{-1}\frac{x^2-y^2}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$.

5. Solve the following system of linear equations by Cramer's Rule or Matrix Inversion method

$$2x + 5y + 3z = 5$$
$$3x + y + 2z = 5$$
$$x + 2y - z = 0$$

6. Verify Rolle's theorem for the function $f(x) = |x-2|, 0 \le x \le 4$.

Group – C

(Long Answer Type Questions)

Answer *any three* of the following. 15×3=45

7. (a) Find the 3rd term from end in the expansion of $\left(x^2 + \frac{1}{2x}\right)^{13}$

(b) Find A^{-1} where $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \\ -2 & 1 & 5 \end{bmatrix}$

(c) Evaluate $\int \frac{x^5\,dx}{x^2+1}$

8. (a) If $\lim_{x\to 0}\frac{ae^x - b}{x} = 2$ then find the value of a, b.

(b) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(c) Find the area bounded by $y = 2 - x^2$ and $x + y = 0$

9. (a) Expand $\log(1+2x)$ using Maclaurin's series.

(b) The parabola $y^2 = 2ax$ passes through the center of the circle $4x^2 + 4y^2 - 8x + 12y - 7 = 0$. Find the focus and length of the latus-rectum of the parabola.

(c) If $y = \frac{3x-1}{(x+3)(x-1)}$, find $\frac{dy}{dx}$.

10. (a) A straight line passes through the point (2,3) and the sum of its intercepts on X-axis and Y-axis is 10. Find the equation of the straight line.

(b) A function $f(x)$ is defined as follows:

$f(x) = x + 1$, when $x \geq 1$

$= \frac{3}{2}$, when $x = 1$

$= x$, when $x < 1$

Draw the graph of $f(x)$ and examine the continuity of $f(x)$ at $x = \frac{1}{2}$.

(c) Find the equation of the circle whose center $\left(\frac{5}{3}, -3\right)$ is and which touches the line $3x + 2y + 5 = 0$.

11. (a) Evaluate $\int_{-2}^{2} |1 - x^2| dx$

(b) Using MVT prove that $1 + \frac{x}{2\sqrt{1+x}} < \sqrt{1+x} < 1 + \frac{x}{2}$

(c) Evaluate $\lim_{n\to\infty}\left[\frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{2n+n}\right]$
