Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.ARCH/SEM-1/ARCH-102/2012-13
2012
MATHEMATICS - I

Time Allotted : 3 Hours   *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

   i)   If $y = ax^n + b$ then $y_{n+1} =$

   a) $n!$ b) $n!\,a$

   c) 0 d) $1.$

   ii) nth derivative of sin ($5x + 3$) is

   a) $5^n \cos (5x + 3)$

   b) $5^n \sin \left(\dfrac{n\pi}{2} + 5x + 3 \right)$

   c) $15 \sin \left(\dfrac{n\pi}{2} + 5x + 3 \right)$

   d) $- \sin (5x + 3).$

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iii) The Rolles theorem is not applicable in $[-1, 1]$ to the function

a) x^2

b) $x^2 + 1$

c) $|x|$

d) none of these.

iv) In the Taylor's series expansion of $f(x)$ the expression $\frac{h^n}{n!} f^{(n)}(a + \theta h)$ is known as

a) Cauchy's remainder

b) Rolles remainder

c) Taylor's remainder

d) Lagrange's remainder.

v) If radius of curvature of a curve at (x_1, y_1) is ρ then what is the curvature of that curve at that point ?

a) $\frac{1}{\rho}$

b) ρ^2

c) 0

d) 1.

vi) Angle between the curves $y = x^2$ & $y = 2x$ is

a) 0°

b) 90°

c) 180°

d) 270°.

vii) The sequence $\{(-1)^n\}$ is

a) convergent

b) oscillatory

c) divergent

d) none of these.

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viii) The series $\Sigma \dfrac{1}{n^p}$ is convergent if

    a) $p \geq 1$ b) $p > 1$

    c) $p < 1$ d) $p \leq 1$.

ix)   Rank of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ is

    a)   0   b)   1

    c)   2   d)   3.

x)   The value of t for which the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is

   singular is

    a) $\dfrac{3}{2}$ b)   2

    c)   1   d) $\dfrac{1}{3}$.

xi)   The equation $x + y + z = 0$ has

    a)   Infinite number of solutions

    b)   No solution

    c)   Unique solutions

    d)   Two solutions.

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GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. State Lagrange's mean value theorem. Show that

$$\frac{x}{1+x} < \log_e (1+x) < x \text{ holds if } x > 0.$$

3. What is D'Alembert's ratio test. Use it to test the following

series is convergent or not.

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \text{............} \infty$$

4. Prove that

$$\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} = 4a^2 b^2 c^2$$

5. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

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GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.   3 × 15 = 45

6. a) If $y = \cos (ax + b)$ then find y_n 3

b) If $y = \log_e x$ and $y = \log_{10} x$ find the angle of intersection

of these two curves at (1, 0).   4

c) Solve the given system of equations by matrix inversion

method and Cramer's rule and veryfy that the solutions

are same.

$2x + 3y + z = 11$

$x + y + z = 6$

$5x - y + 10z = 34.$ 8

7. a) Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \frac{x^4}{17} + \frac{x^5}{26} + \ldots\ldots\ldots \infty, \ (x > 0).$$ 7
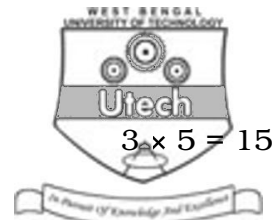
b) Find all the asymptotes of

$xy^2 - y^2 - x^3 = 0.$

OR

Find the envelope of the family of co-axial ellipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, given that $a^n + b^n = c^n$.   8

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8. Answer any *three* : $3 \times 5 = 15$

    a) State Taylor's theorem with Lagrange form of remainder and Cauchy form of remainder.

    b) State Rolles theorem and verify Rolles theorem for $f(x) = |x|$ in $-1 \le x \le 1$.

    c) Prove that the series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots + (-1)^{n+1} \frac{x^n}{n} + \ldots\ldots \infty \text{ is}$$

       convergent when $|x| < 1$.

    d) Prove that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8\,abc.$

9. a) Determine the conditions under which the system of equations

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2$$

       admits only one solution, no solution and many solutions. 8

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b) Find the envelope of $\dfrac{x}{a} + \dfrac{y}{b} = 1$

Given that $a^2 + b^2 = c^2$. 4

c) Give the geometrical interpretations of Lagrange's Mean value theorem. 3

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