

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/INTPBIR/SEM-3/PHY-302/2009-10**

**2009**

**CONDENSED MATTER PHYSICS**

Time Allotted : 3 Hours

Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

*Answer all questions.*

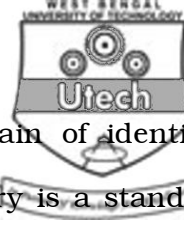
1. a) Two identical masses, attached by springs to each other and to the rigid walls, as shown, are constrained to move only in the  $x$ -direction.

Set up the equations of motion and look for solutions of the form.

$$U_1 = A_1 \exp(i\omega t) \text{ and } U_2 = A_2 \exp(i\omega t)$$

Justify the above solutions and find the allowed frequencies.

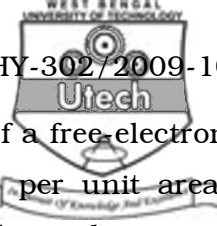
4



- b) Show that in the case of a linear chain of identical atoms the solution at the zone boundary is a standing wave and not a travelling one. 2
- c) The lattice specific heat of a certain form of carbon has a  $T^2$  temperature dependence instead of the more usual  $T^3$  dependence for solids. What can you infer about the structure of this phase of carbon ? Give detailed reasoning. 2
2. Consider the propagation of elastic waves in a tetragonal crystal (  $a = b \neq c$ ,  $\alpha = \beta = \gamma = \pi/2$  ). Show that for  $K$  parallel to (i)  $a$ -axis and (ii)  $c$ -axis, pure longitudinal and pure transverse waves may be propagated. What are the velocities and polarization directions of these waves ? Why are the transverse waves degenerate ( same velocity ) in (ii) and not in (i) ? 8

Given :  $\partial P_{ij} / \partial X_j = \rho \partial^2 U_i / \partial t^2$  and

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{11} & C_{13} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & C_{66} \end{pmatrix}$$



3. a) Prove that the density of states  $N(E)$  of a free-electron gas in two dimension (  $2D$  ) is  $m/\pi\hbar^2$  per unit area. Using  $N(E)$  for both electrons and holes with masses  $m_e^*$  and  $m_h^*$ , derive expressions for carrier concentrations of both types vs  $T$  for a  $2D$  intrinsic semiconductor. Find the position of the Fermi level. 6
- b) A three dimensional intrinsic semiconductor has a resistivity of  $1.21 \Omega \text{ cm}$  at  $27^\circ\text{C}$  and  $1.10 \Omega \text{ cm}$  at  $57^\circ\text{C}$ . Estimate the energy gap assuming that the change in mobility in this temperature range is negligible. Given  $k_B = 1.38 \times 10^{-16} \text{ ergs/K} = 1.38 \times 10^{-23} \text{ J/K}$ . 2
4. a) Using the free-electron model of a solid, show explicitly that the electrical resistance does not depend on the magnetic field while the Hall resistance is linearly proportional to it.

You may find the following equations helpful :

$$\hbar[(d/dt) + (1/\tau)]\delta k = F$$

$$\delta v_x = \left[ (-e\tau/m) / (1 + (\omega_c\tau)^2) \right] [E_x - \omega_c\tau E_y],$$

$$\delta v_y = \left[ (-e\tau/m) / (1 + (\omega_c\tau)^2) \right] [E_y - \omega_c\tau E_x] \text{ and}$$

$$\delta v_z = (-e\tau/m)[E_z],$$

where  $\omega_c = eH_z/mc$  ( CGS units ). 8

- b) Write in a few lines the magnetic field dependence of resistivity of a metal at low and high fields in the two-band model. 2



5. a) It is found that in a structural phase transition from the *bcc* to an *fcc* structure, a particular metal does not change its volume. Find the ratio of the nearest neighbour distance in the two structures. 2

- b) In a two electron system like hydrogen molecule, what are the spin configurations of the singlet and the triplet states ? Show that the exchange Hamiltonian is given by

$$H_e = -2 j_{ij} S_i \cdot S_j. \quad 6$$

6. a) What is the order of the superconducting phase transition ? Justify your answer with solid reasoning and quantitative details. Prove that the difference between the specific heats in the normal and the superconducting states is given by

$$C_n - C_s = V H_0^2 / 2\pi T_c \left[ (T/T_c) - 3(T/T_c)^3 \right]. \quad 6$$

Hint : Free energy  $F ( T, H_c )$  is continuous along  $H_c (T) = H_0 \left[ 1 - (T/T_c)^2 \right]$  curve.

- b) Why are good metals not superconductors ? 2

Hint :  $T_c$  of BCS theory.

=====