Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/INTPBIR/SEM-3/PHY-302/2009-10

2009
CONDENSED MATTER PHYSICS

Time Allotted : 3 Hours Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer *all* questions.

1. a) Two identical masses, attached by springs to each other and to the rigid walls, as shown, are constrained to move only in the x-direction.

Set up the equations of motion and look for solutions of the form.

$U_1 = A_1 \exp(i\omega t)$ and $U_2 = A_2 \exp(i\omega t)$

Justify the above solutions and find the allowed frequencies. 4

b) Show that in the case of a linear chain of identical atoms the solution at the zone boundary is a standing wave and not a travelling one. 2

c) The lattice specific heat of a certain form of carbon has a T^2 temperature dependence instead of the more usual T^3 dependence for solids. What can you infer about the structure of this phase of carbon ? Give detailed reasoning. 2

2. Consider the propagation of elastic waves in a tetragonal crystal ($a = b \neq c$, $\alpha = \beta = \gamma = \pi/2$). Show that for K parallel to (i) a-axis and (ii) c-axis, pure longitudinal and pure transverse waves may be propagated. What are the velocities and polarization directions of these waves ? Why are the transverse waves degenerate (same velocity) in (ii) and not in (i) ? 8

Given : $\partial P_{ij} / \partial X_j = \rho \partial^2 U_i / \partial t^2$ and

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{11} & C_{13} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & C_{66} \end{pmatrix}$$

3. a) Prove that the density of states $N(E)$ of a free-electron gas in two dimension ($2D$) is $m/\pi\hbar^2$ per unit area. Using $N(E)$ for both electrons and holes with masses m_e^* and m_h^*, derive expressions for carrier concentrations of both types *vs* T for a $2D$ intrinsic semiconductor. Find the position of the Fermi level. 6

b) A three dimensional intrinsic semiconductor has a resistivity of 1·21 Ω cm at 27°C and 1·10 Ω cm at 57°C. Estimate the energy gap assuming that the change in mobility in this temperature range is negligible. Given $k_B = 1{\cdot}38 \times 10^{-16}$ ergs/K $= 1{\cdot}38 \times 10^{-23}$ J/K. 2

4. a) Using the free-electron model of a solid, show explicitly that the electrical resistance does not depend on the magnetic field while the Hall resistance is linearly proportional to it.

You may find the following equations helpful :

$$\hbar\left[(\mathrm{d}/\mathrm{d}t) + (1/\tau)\right]\delta k = F$$

$$\delta v_x = \left[(-e\,\tau/m)/\left(1+(\omega_c\tau)^2\right)\right]\left[E_x - \omega_c\,\tau\,E_y\right],$$

$$\delta v_y = \left[(-e\,\tau/m)/\left(1+(\omega_c\tau)^2\right)\right]\left[E_y - \omega_c\,\tau\,E_x\right] \text{ and}$$

$$\delta v_z = (-e\,\tau/m)\left[E_z\right],$$

where $\omega_c = eH_z/mc$ (CGS units). 8

b) Write in a few lines the magnetic field dependence of resistivity of a metal at low and high fields in the two-band model. 2

5. a) It is found that in a structural phase transition from the *bcc* to an *fcc* structure, a particular metal does not change its volume. Find the ratio of the nearest neighbour distance in the two structures. 2

b) In a two electron stystem like hydrogen molecule, what are the spin configurations of the singlet and the triplet states ? Show that the exchange Hamiltonian is given by

$H_e = -2\, j_{ij}\, S_i \,.\, S_j$. 6

6. a) What is the order of the superconducting phase transition ? Justify your answer with solid reasoning and quantitative details. Prove that the difference between the specific heats in the normal and the superconducting states is given by

$Cn - Cs = VH_0^2 / 2\pi T_C \left[(T/T_C) - 3(T/T_C)^3 \right]$. 6

Hint : Free energy F (T, Hc) is continuous along $H_C(T) = H_0 \left[1 - (T/T_C)^2 \right]$ curve.

b) Why are good metals not superconductors ? 2

Hint : T_C of BCS theory.