Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/Int.PBR(PHY)/SEM-3/PHY-301/2010-11

2010-11

QUANTUM MECHANICS - III

Time Allotted : 3 Hours *Full Marks* : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* of the following. $5 \times 10 = 50$

1. a) For the Dirac electron in a central potential show that $\vec{L} = \vec{r} \times \vec{p}$ is not a constant of motion but $\vec{L} + \frac{1}{2} \hbar \vec{\Sigma}$. What is $\vec{\Sigma}$? Argue that this corresponds to an intrinsic spin of $\frac{1}{2}$.

   b) Show that the only component of $\vec{\Sigma}$ that commutes with the Dirac Hamiltonian of a free particle is the one along the momentum $\vec{p}$. 10

2. Let $\psi(x)$ and $\psi'(x')$ be the Dirac wave functions in frames K and K' respectively.

   a) Consider $\psi(x)$ and $\psi'(x')$ to be related by the linear transformation $\psi'(x') = S(\Lambda)\psi(x)$, where Λ denotes the Lorentz transformation. Derive the conditions on S if the Dirac equation is to preserve its form in the two frames.

b) When Λ denotes a space inversion, verify that the condition derived in (a) above is satisfied if one chooses $S(\Lambda) = \gamma^0$. 10

3. Consider a no-nrelativistic spin $\frac{1}{2}$ particle.

a) Argue that one can postulate that the time reversal operator T must obey the condition $T\vec{S}\,T^{-1} = -\vec{S}$.

b) Given that the form $T = -i\sigma_y K$ (where K is the complex conjugation operator) satisfies the above condition. Show that $T^2 = -1$.

c) Show that for a system of N spin $\frac{1}{2}$ particle $T^2 = (-1)^N$ and argue how this fact leads to Kramer's degeneracy.

10

4. The quantized electromagnetic field in Coulomb gauge is described by the vector potential

$$A(\vec{r}, t) = \sum_{\vec{k}\lambda} \sqrt{\frac{2\pi\, c}{Vk}}\; \hat{e}_{\vec{k}\lambda}\, a_{\vec{k}\lambda}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} + H.c.$$

Generalize the classical definition of the momentum of the field, *viz.*,

$$\vec{P}_{cl} = \frac{1}{4\pi c} \int_V d^3r\, \vec{E} \infty \vec{B},$$

to the quantum domain. Show that the momentum operator can be expressed in the form,

$$\vec{P} = \sum_{\vec{k}\lambda} \vec{k}\, a^{\dagger}_{\vec{k}\lambda}\, a_{\vec{k}\lambda}$$ 10

5. Consider a single mode of the quantized electromagnetic field.

   a) Calculate the photon number distribution, the mean and variance of the number operator in the coherent state.

   b) Argue how the coherent state allows one to make a transition from the quantum to the classical description of the electromagnetic fields. 10

6. Consider a non-relativistic free electron interacting with the quantized electromagnetic field via the interaction Hamiltonian $H_1 = -\frac{e}{mc}\vec{A}\cdot\vec{p}$, where $\vec{p}$ denotes the momentum operator of the electron. Consider the field to be in the vacuum state.

   a) Calculate the energy shift of the electron in the state $|\vec{p}\rangle$ due to the interaction, using second order perturbation theory.

   b) Sum over all final states by going to the continuum limit and show that this shift is linearly divergent. 10

7. Consider an atom (in the excited state $|e\rangle$ interacting with the quantized electromagnetic field (in the vacuum state) and undergoing spontaneous emission to the ground state $|g\rangle$. Consider the eletric dipole interaction Hamiltonian $H_1 = -\vec{d} \cdot \vec{E}$.

   a) Use the Fermi golden rule to calculate the spontaneous emission rate.

   b) Obtain an explicit expression by summing over all final states in the continuum limit. 10