



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/Int.PBR(PHY)/SEM-3/PHY-301/2010-11**

**2010-11**

**QUANTUM MECHANICS - III**

Time Allotted : 3 Hours

Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any *five* of the following.

5 × 10 = 50

1. a) For the Dirac electron in a central potential show that  $\vec{L} = \vec{r} \times \vec{p}$  is not a constant of motion but  $\vec{L} + \frac{1}{2} \hbar \vec{\Sigma}$ . What is  $\vec{\Sigma}$ ? Argue that this corresponds to an intrinsic spin of  $\frac{1}{2}$ .  
 b) Show that the only component of  $\vec{\Sigma}$  that commutes with the Dirac Hamiltonian of a free particle is the one along the momentum  $\vec{p}$ . 10
2. Let  $\psi(x)$  and  $\psi'(x')$  be the Dirac wave functions in frames  $K$  and  $K'$  respectively.
  - a) Consider  $\psi(x)$  and  $\psi'(x')$  to be related by the linear transformation  $\psi'(x') = S(\Lambda) \psi(x)$ , where  $\Lambda$  denotes the Lorentz transformation. Derive the conditions on  $S$  if the Dirac equation is to preserve its form in the two frames.



- b) When  $\hat{A}$  denotes a space inversion, verify that the condition derived in (a) above is satisfied if one chooses  $S(\hat{A}) = \gamma^0$ . 10

3. Consider a non-relativistic spin  $\frac{1}{2}$  particle.

- a) Argue that one can postulate that the time reversal operator  $T$  must obey the condition  $T S T^{-1} = -S$ .
- b) Given that the form  $T = -i\sigma_y K$  (where  $K$  is the complex conjugation operator) satisfies the above condition. Show that  $T^2 = -1$ .
- c) Show that for a system of  $N$  spin  $\frac{1}{2}$  particles  $T^2 = (-1)^N$  and argue how this fact leads to Kramer's degeneracy.

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4. The quantized electromagnetic field in Coulomb gauge is described by the vector potential

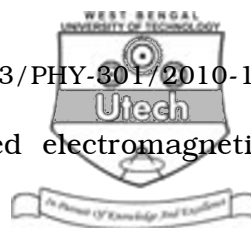
$$A(\mathbf{r}, t) = \sum_{\mathbf{k}\lambda} \sqrt{\frac{2\pi c}{V k}} \hat{e}_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + H.c.$$

Generalize the classical definition of the momentum of the field, viz.,

$$\mathbf{P}_{cl} = \frac{1}{4\pi c} \int_V d^3r \mathbf{E} \times \mathbf{B},$$

to the quantum domain. Show that the momentum operator can be expressed in the form,

$$\mathbf{P} = \sum_{\mathbf{k}\lambda} \mathbf{k} a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} \quad \text{10}$$



5. Consider a single mode of the quantized electromagnetic field.

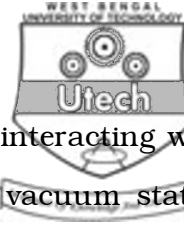
- a) Calculate the photon number distribution, the mean and variance of the number operator in the coherent state.
- b) Argue how the coherent state allows one to make a transition from the quantum to the classical description of the electromagnetic fields.

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6. Consider a non-relativistic free electron interacting with the quantized electromagnetic field via the interaction Hamiltonian  $H_1 = -\frac{e}{mc} \vec{A} \cdot \vec{p}$ , where  $\vec{p}$  denotes the momentum operator of the electron. Consider the field to be in the vacuum state.

- a) Calculate the energy shift of the electron in the state  $|\vec{p}\rangle$  due to the interaction, using second order perturbation theory.
- b) Sum over all final states by going to the continuum limit and show that this shift is linearly divergent.

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7. Consider an atom ( in the excited state  $|e\rangle$  interacting with the quantized electromagnetic field ( in the vacuum state ) and undergoing spontaneous emission to the ground state  $|g\rangle$ . Consider the electric dipole interaction Hamiltonian  $H_1 = -\vec{d} \cdot \vec{E}$ .

- a) Use the Fermi golden rule to calculate the spontaneous emission rate.
- b) Obtain an explicit expression by summing over all final states in the continuum limit.

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