



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/INT.PBIR(PHY)/SEM-2/PHY-203/2010**

**2010**

**QUANTUM MECHANICS - II**

Time Allotted : 3 Hours

Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any *five* questions.

All questions carry equal marks.

1. For a 2-dimensional simple harmonic oscillator acted upon by a perturbation  $V(x, y) = \alpha xy$ , find the lowest order non-vanishing corrections to the energies of the ground state and the first excited state. Compare with the exact results.

2. For a hydrogen atom in a constant electric field

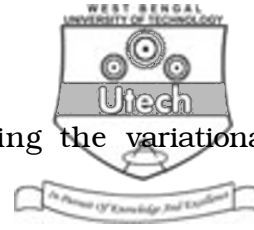
**Error!**

3. Using  $[J^2, [J^2, V_i]] = 2\hbar^2 (J^2 V_i + V_i J^2) - 4\hbar^2 (\vec{V} \cdot \vec{J}) J_i$

for any vector operator  $V_i$ , show that

$$\langle jm' | V_i | jm \rangle = \langle j || V^{(1)} || j \rangle \langle jm' | J_i | jm \rangle$$

for some constant  $\langle j || V^{(1)} || j \rangle$  independent of  $m, m'$ .



4. For the Helium atom ground state, using the variational wavefunction

$$\psi(r_1, r_2) = \psi_{100}(r_1) \psi_{100}(r_2)$$

$$\text{with } \psi_{100}(r) = \frac{2}{\sqrt{4\pi}} \left( \frac{z}{a_0} \right)^{3/2} e^{-zr/a_0}$$

find the expectation value of the interaction energy

$$\frac{e^2}{|r_1 - r_2|}.$$

5. For transitions from an initial state  $|i\rangle$  at time  $t \rightarrow -\infty$  to a final state  $|f\rangle$  at time  $t \rightarrow +\infty$  derive Fermi's golden rule for the rate of transition

$$R_{i \rightarrow f} = \frac{2\pi}{\hbar} \delta(E_f - E_i) | \langle f | H_I | i \rangle |^2$$

where  $|i\rangle$ ,  $|f\rangle$  are eigenstates of the free Hamiltonian and  $H_I$  does not depend explicitly on time.

6. For scattering from a "hard sphere" potential

$$V(r) = 0, r > a$$

$$= \infty, r < a$$

find an expression for the partial wave phase shift  $\delta_l$ . Show that for some values of the incident energy, the  $l = 0$  states do not "see" the sphere. Calculate  $\delta_l$  in the limits of very high and very low incident energies.



7. For scattering from a potential

$$V = V_0, r < a$$

$$= 0, r > a$$

Calculate the phase shift  $\delta_0$ . Considering  $|V_0| \ll E$  relate the sign of the phase shift to the potential being attractive or repulsive.

8. Given two electrons in a constant magnetic field, ignore all other dynamics but the spin-magnetic interaction. Write the canonical density matrix  $\rho$  and the thermodynamic entropy  $S = -k \text{Tr} \rho \log \rho$  where  $k$  is the Boltzmann constant. Calculate  $\langle H \rangle$  by explicitly evaluating  $\text{Tr}(\rho H)$  and also from  $-\frac{\partial}{\partial \beta} \text{Tr}(e^{-\beta H})$ .

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