Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/INT.PBIR(PHY)/SEM-2/PHY-203/2010

2010

QUANTUM MECHANICS - II

Time Allotted : 3 Hours Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions.

All questions carry equal marks.

1. For a 2-dimensional simple harmonic oscillator acted upon by a perturbation $V(x, y) = \alpha xy$, find the lowest order non-vanishing corrections to the energies of the ground state and the first excited state. Compare wtih the exact results.

2. For a hydrogen atom in a constant electric fiedl **Error!**

3. Using $[J^2, [J^2, V_i]] = 2\hbar^2(J^2 V_i + V_i J^2) - 4\hbar^2 (\vec{V}\cdot\vec{J})\, J_i$

   for any vector operator V_i, show that

   $< jm' \mid V_i \mid jm > = < j \,||\, V^{(1)} \,||\, j > < jm' \mid J_i \mid jm >$

   for some constant $< j \,||\, V^{(1)} \,||\, j >$ independent of m, m'.

4. For the Helium atom ground state, using the variational wavefunction

$$\psi(r_1, r_2) = \psi_{100}(r_1)\ \psi_{100}(r_2)$$

with $\psi_{100}(r) = \frac{2}{\sqrt{4\pi}}\left(\frac{z}{a_0}\right)^{3/2} e^{-z_r/a_0}$

find the expectation value of the interaction energy

$$\frac{e^2}{|r_1 - r_2|}.$$

5. For transitions from an initial state $|i>$ at time $t \to -\infty$ to a final state $|f>$ at time $t \to +\infty$ derive Fermi's golden rule for the rate of transition

$$R_{i\to f} = \frac{2\pi}{\cdot}\ \delta(E_f - E_i)\ |<f\ |\ H_I\ |\ i>|^2$$

were $|i>$, $|f>$ are eigenstates of the free Hamiltonian and H_I does not depend explicitly on time.

6. For scattering from a 'hard sphere" potential

$$V(r) = 0,\ r > a$$

$$= \infty,\ r < a$$

find an expression for the partial wave phase shift δ_l. Show that for some values of the incident energy, the $l = 0$ states do not "see" the sphere. Calculate δ_l in the limits of very high and very low incident energies.

7. For scattering from a potential

$$V = V_0 \ , \ r < a$$

$$= 0 \ , \quad r > a$$

Calculate the phase shift δ_0. Considering $| V_0 | << E$ relate the sign of the phase shift to the potential being attractive or repulsive.

8. Given two electrons in a constant magnetic field, ignore all other dynamics but the spin-magnetic interaction. Write the canonical density matrix ρ and the thermodynamic entropy $S = - k \ Tr \ \rho \log \rho$ where k is the Boltzmann constant. Calculate $< H >$ by explicitly evaluating $Tr (\rho H)$ and also from $- \frac{\partial}{\partial \beta} Tr (e^{-\beta H})$.