Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/INTPBIR/SEM-1/PHY-101/2009-10

2009

CLASSICAL MECHANICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. Show that for Keplerian motion governed by the force

$$\vec{F} = -\left(\frac{mk}{r^3}\right)\vec{r}$$

the Runge-Lenz vector

$$\vec{A} = \vec{p} \times \vec{L}/m - k\vec{r}/r$$

(with $\vec{L} = \vec{r} \times \vec{p}$ the angular momentum) is a constant of motion. Discuss the physical significance of this vector.

2. Find the Lagrangian and Hamiltonian for a charged particle of charge e and mass m moving in a uniform time independent magnetic field $\vec{B}$.

3. If the Lagrangian of a particle moving in three dimensions is

   i) invariant under rotations about the Z-axis prove that the component of the angular momentum along the Z-axis is conserved.

   ii) invariant under translations along the X-axis prove that the momentum along the X-axis is a constant of the motion.

4. Discuss schematically how you would go about determining the amplitude dependence of the time period of a simple pendulum governed by the equation

$$\frac{d^2\theta}{dt^2} = -(g/l)\sin\theta$$

   when the angular amplitude is not small enough to allow you to approximate $\sin\theta$ by θ.

5. Make a linear stability analysis for the system governed by the equations

$$\dot{x} = x - xy$$

$$\dot{y} = -y + xy$$

   and sketch qualitatively the phase portrait.

6. Consider an infinite chain of point massees (each of mass m) joined to each other by ideal weight-less springs each of relaxed length Δ and spring constant k.

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If the mases move only longitudinally along the chain, write down the Newton's equation for the displacement $\xi_n(t)$ of the n^{th} point mass. Explain carefully the limiting procedure which leads to the equation for longitudinal waves along a continuous string of mass per unit length ρ and tension T governed by the equation

$$\rho \frac{\partial^2 \sum(x, t)}{\partial t^2} = T \frac{\partial^2}{\partial x^2} \xi(x, t)$$

and write down the Lagrangian density corresponding to this system.

7. Consider a triatomic molecule ABC with atoms A & B of mass m_A and m_B which is linear and confine your attention to motion along that line. Set up the Lagrangian describing the linear vibrations of this system assuming that the forces between the atoms A & B can be approximated by linear proportionality to the stretch of the bonds. Determine the normal modes of vibration and describe the corresponding motion of the atoms.

8. Given that the motion of a particle is governed by the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2} m w^2 x^2 + \lambda xp$$

where m, w and λ are constants, find the canonical transformation which brings the Hamiltonian to the form

$$AP^2 + BX^2$$

where P & X are the new momentum and coordinate.