Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/Int.PBR(PHY)/SEM-1/PHY-101/2010-11

2010-11

CLASSICAL DYNAMICS

Time Allotted : 3 Hours *Full Marks* : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *two* from the rest.

1. a) Derive the expression for the general variation, δS of the action S of a system with n degrees of freedom when the Lagrangian

$$L \int L\left(q, \dot{q}, t\right), q = q_1, q_2, \ldots\ldots q_n. \qquad 8$$

b) Obtain Noether's identity. 4

c) Deduce Lagrange's equation of motion from Hamilton's principle of least action. 4

d) Consider a system of N particles given by the Lagrangian :

$$\left[L = \frac{1}{2} \sum_{a=1}^{N} m_a \dot{r}_a^{\,2} - \sum_{a \neq b} V\left(\left| r_a - r_b \right|\right). \right]$$

Deduce the conserved quantity using the result (b) above when the action has a symmetry under Galilean transformation. 6

2. a) Show that in a rigid body rotation, a particle fixed in the body with a position vector r is carried to a position s given by :

$$s = \gamma r + (1 - \gamma) \; (n \cdot r) \, n + \sigma n \wedge r$$

n being the unit vector along the axis of rotation and $\gamma^2 + \sigma^2 = 1$. 7

b) Determine the form of a curve such that the frequency of oscillations of a particle on it under the force of gravity is independent of the amplitude. 7

3. a) Show that in the nonrelativistic Kepler problem with the Lagrangian $L = \frac{1}{2} \dot{x}_j \dot{x}_j + \frac{K}{r}$, the Lenz vector $\wedge = V \wedge L - \frac{K}{r} r$ is a constant of motion. Describe the symmetry corresponding to this. 7

b) A system of two degrees of freedom is described by the Hamiltonian :

$H = q_1 p_1 - q_2 p_2 - a \, q_1^{\,2} + b \, q_2^{\,2}$, where a and b are constants. Show that $F_1 = \dfrac{p_1 - a\, q_1}{q_2}$, $F_2 = q_1$ q_2 are constants of motion. Are there any other independent algebraic constants of motion ? Can any be constructed from the Jabcobi identity ? 7

4. a) Using Euler's angles construct the Lagrangian for a spinning top and write down the equations of motion. 7

b) Show that the spin Ω of a spinning top in its steady motion must be a root of the quadratic equation :

$$F(\Omega) \int \cos\alpha\, \Omega^2 - 2p\,\Omega + q = 0$$

where α is the inclination of the axis of the top to the z-axis and p, q are determined by the principal moments of inertia at the peg. 7

5. a) Show that the infinitesimal change of any function f of q and p caused by an infinitesimal canonical transformation with generator $F(q, p)$ and parameter □ is given by $\delta f(q, p) = \{ f(q, p), \square F \}$. What do you infer if $F \int H$? 7

b) Prove that the transformation

$$Q_1 = q_1^2, \; Q_2 = q_2 \sec p_2, \; P_1 = \frac{p_1 \cos p_2 - 2q_2}{2q_1 \cos p_2},$$

$P_2 = \sin p_2 - 2q_1$ is canonical. 7