Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(VLSI)/SEM-1/MVLSI-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Question No. 1 is compulsory. Answer any *four* from the rest.

$$5 \times 14 = 70$$

1. a) If A and B are two independent events then show that A and B^c are also independent. 4

    b) If $f(x, y) = \sqrt{|xy|}$ show that

$$f_x(x, y) = \begin{cases} \dfrac{1}{2}\sqrt{\dfrac{|y|}{|x|}} & \text{if } x > 0 \\[3mm] -\dfrac{1}{2}\sqrt{\dfrac{|y|}{|x|}} & \text{if } x < 0 \end{cases}$$

$$\text{and } f_y(x, y) = \begin{cases} \dfrac{1}{2}\sqrt{\dfrac{|x|}{|y|}} & \text{if } y > 0 \\[3mm] -\dfrac{1}{2}\sqrt{\dfrac{|x|}{|y|}} & \text{if } y < 0 \end{cases} \qquad 5$$

    c) Evaluate $\displaystyle\int_c \frac{dz}{(z-a)^n}$ for $n = 2, 3, 4, \ldots\ldots$ where c is a closed curve containing the point $z = a$. 5

40504   [Turn over

2. a) Examine the maxima and minima of the function
$f(x, y) = 2x^2 - xy + 2y^2 - 20x$. 7

b) Let $y = F(x, t)$, where F is a differentiable function of two independent variables x and t which are related to variables u and v by the relations $u = x + ct$, $v = x - ct$.

Prove that $\dfrac{\partial^2 y}{\partial^2 x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial^2 t^2} = 0$ can be transformed

into $\dfrac{\partial^2 y}{\partial u\, \partial v} = 0$ 7

3. a) Find the stationary points of $f(x, y, z) = x^2\, y^2\, z^2$ subject to the condition $x^2 + y^2 + z^2 = a^2$, where x, y, z are positive. Also, find the maximum value of the same function subject to the condition $x^2 + y^2 + z^2 = a^2$ by application of Lagrangian multiplier method. 7

b) If $f(0) = 0$ and $f'(x) = \dfrac{1}{1 + x^2}$ then prove without using method of integration that $f(x) + f(y) = $ **Error!**)

7

4. a) The value of $\sin x$ for different values of x are given below. Form a difference table and from this table find $\sin 32°$ and $\sin 53°$ using proper formula. 7

$x°$	30	35	40	45	50	55
$\sin x$	0·5000	0·5736	0·6428	0·7071	0·7660	0·8192

b) Find the polynomial $f(x)$ and hence calculate $f(5·5)$ for the given data : 7

x	0	2	3	4	7
$f(x)$	1	47	97	251	477
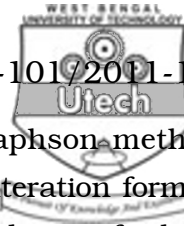
5. a) Find the convergence of the Newton-Raphson method. Using Newton-Raphson method, obtain iteration formula for the reciprocal of a number N and hence find the value of $\dfrac{1}{22}$, correct to three significant figures.   7

    b) Using the modified Euler's method find y (1·2) where $\dfrac{dy}{dx} = \dfrac{x + y}{2}$, y (1) = 3·595 and h = 0·1.   7

6. a) Expand $f (z) = \dfrac{1}{z^2 (z - i)}$ as a Laurent's series about i and hence find the residue there.   7

    b) $\displaystyle\int_0^{2\pi} \dfrac{1 + \sin\theta}{3 + \cos\theta} \, d\theta$ using method of residues.   7

7. a) Evaluate $\displaystyle\int_0^{2+i} (\bar{z})^2 \, dz$ along the following paths   7

      i) the straight line $y = \dfrac{x}{2}$

      ii) first along the real axis to 2 and then vartically to (2 + i)

    b) Determine the analytic function whose real part is $e^x (x \cos y - y \sin y)$.   7

8. a) X and Y stand in a queue at random with 10 other people. What is the probability that there are exactly 3 people between X and Y ?   7

    b) There are 3 good and 1 bad coins. The bad one has head on both sides. A coin is chosen randomly and tossed 4 times. If head occurs all the 4 times what is the probability that the bad coin has been chosen for toss ?   7

9. a) If the daily wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation of Rs. 70 and Rs. 5 respectively, find the expected number of workers whose daily wages are :

    i)   between Rs. 66 and Rs. 72

    ii) more than Rs. 72.

    Here it is given that : $\dfrac{1}{\sqrt{2\,\pi}} \displaystyle\int_{0}^{0 \cdot 4} e^{-t^2/2}\ dt = 0 \cdot 1554$

    and

    $\dfrac{1}{\sqrt{2\,\pi}} \displaystyle\int_{0}^{0 \cdot 8} e^{-t^2/2}\ dt = 0 \cdot 2881.$ 7

    b) Let X denote the number of misprints on a page in a certain book. Assume that the random variable X follows Poisson distribution. If $E(X^2) = 6$ then find out the probability that a randomly chosen page will have at least one misprint.   7

═══════