BName : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(VLSI)/SEM-1/PGMVD-101/2012-13

2012
ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks : 70*

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Objective Type Questions)

1. Answer any *five* questions: $5 \times 2 = 10$

   a) Find the Lalace transform of

   $$f(t) = 1 + 2t - \frac{1}{3}t^4$$

   b) Find the inverse Laplace transform of $F(s) = \dfrac{4s}{s^2 - 16}$.

   c) For an even function of period 2L, write down the expressions for the Fourier coefficients a_0 and a_n.

   d) Write down the Fourier transform of $f''(x)$ in terms of the Fourier transform of $f(x)$ and ω.

   e) Write the necessary and sufficient condition for a graph to posses an Euler trail.

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f)   Define a bipartite graph and a complete bipartite graph illustrating by diagrams.

g)   Define isomorphism of graphs. Draw two graphs which are isomorphic.

h)   What do you mean by chromatic polynomial of a graph ? Write the chromatic polynomial of a null graph with n vertices.

GROUP – B

(Long Answer Type Questions)

Answer any *six* of the following. $6 \times 10 = 60$

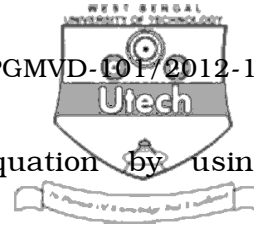2. a)   Find the Laplace transform of the following function

$$f(t) = \begin{cases} 2 & if \quad 0 < t < \pi \\ 0 & if \quad \pi < t < 2\pi \\ \sin t & if \qquad t > 2\pi \end{cases}$$

b)   Determine the inverse Laplace transform of the function

$$F(s) = ln\left(1 + \frac{\omega^2}{s^2}\right).$$ 5 + 5

3. a)   Use the convolution property of the Laplace transform to solve the integral equation :

$$f(t) = e^{-t} + \int_0^t f(t - \tau)\, d\tau.$$

b) Solve the following differential equation by using Laplace transform :

$$\frac{d^2 y}{dt^2} + 16y = 1 + t$$

$y(0) = -2$, $y'(0) = 1$. 5 + 5

4. a) Expand $f(x) = \cos x$ as half-range Fourier sine series in the range $0 \le x \le \pi$.

b) Find the Fourier sine integral representation of the function $f(x) = \sin x$, if $0 \le x \le \pi$

$$= 0, \quad \text{if } x > \pi. \qquad 5 + 5$$

5. a) Find the Fourier cosine and sine transform of the function $f(x) = k$, if $0 < x < a$

$$= 0, \quad \text{if } x > a.$$

b) Calculate the Fourier transform of

$$f(x) = xe^{-x^2}.$$

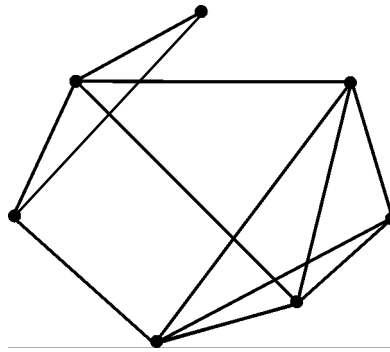Given that $FT\left(e^{-x^2}\right) = \frac{1}{\sqrt{2}} e^{\frac{-\omega^2}{4}}$. 6 + 4

6. a) Prove the necessary and sufficient condition that a given connected graph is semi-Eulerian.

b) Examine whether a simple graph exists for the degree-sequence (1, 1, 3, 3, 3, 4, 5). If it exists, draw it. 6 + 4

7. a) Define planar graph.

b) Draw a 4-vertex complete graph and show that it is self-dual.

c) Examine whether the following graph is planar.

2 + 3 + 5

8. State and prove five-colour theorem. 10

9. a) Prove that if G^I is a subgraph of G, then $X(G^I) \le X(G)$.

b) Prove that a graph with one or more edges is 2-chromatic iff it has no cycle of odd degree.

c) Find he chromatic polynomial of C_4. 2 + 6 + 2