Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(SE)/SEM-2/SE-203/2013

2013

THEORY OF ELASTICITY & PLASTICITY

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

Answer any *four* questions. $4 \times 5 = 20$

1. What do you mean by plane stress condition ? Write down the constitutive relations for a body which is in a state of plane stress.
2. A trial function $\phi = -cxy$ satisfies the bi-harmonic equation $\nabla^2 \nabla^2 \phi = 0$. Show that this potential function represents a case of pure shear.
3. With reference to Cartesian coordinate system the components of the stress (in MPa) tensor at a point in a continuous body are

$$\sigma = \begin{bmatrix} 20 & 40 & 30 \\ 40 & 0 & 5 \\ 30 & 5 & -10 \end{bmatrix}$$

Determine the traction vector $\vec{t}$ and its normal and tangential components at a point on the plane $x + 2y + 3z = 0$, passing through the point.

4. Derive the equilibrium equation in 2-dimensional Cartesian coordinate system.

5. What do you mean by J_2 plasticity ?

GROUP – B

Answer any *one* question. $1 \times 20 = 20$

6. For a cantilever narrow beam (width h and depth d) subjected to end load P, using Airy's stress function find the expression for the stress components as well as for the displacement components within the beam.

7. a) Construct a suitable functional whose Euler-Lagrange equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ in } 0 \le x,\ y \le 1$$

subject to the boundary conditions $u = 0$ on $x = 0, 1$; $y = 0$ and $u = \sin \pi x$ on $y = 1$.

b) Assume $\phi_0(x, y) = y \sin \pi x$, $\phi_1(x, y) = \sin \pi x \sin \pi y$, $\phi_2(x, y) = \sin \pi x \sin 2\pi y$. Using Ritz method find the values of c_1, c_2 and u^h, where $u^h(x, y) = \phi_0(x, y) + c_1\phi_1(x, y) + c_2\phi_2(x, y)$.

GROUP – C

Answer any *two* questions. $2 \times 15 = 30$

8. Show that the following stress components satisfy the equation of equilibrium with zero body force, but are not solution to a problem of elasticity :

$$\sigma = c[y^2 + \nu(x^2 - y^2)], \sigma_y = c[x^2 + \nu(y^2 - x^2)]$$

$$\sigma_x = c\nu(x^2 + y^2), \tau_{xy} = -2\nu xy, \tau_{yz} = \tau_{xz} = 0$$

9. Find the value of torsional constant for a beam with elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are semi-axes respectively.

10. Functional 1 : $\Pi_c(u, N) = -\frac{1}{2}\int_0^L \frac{N^2 \mathrm{d}x}{AE} + \int_0^L N \frac{\partial u}{\partial x} \mathrm{d}x - \int_0^L up\,\mathrm{d}x$

Functional 2 : $\Pi(u) = \int_0^L \frac{1}{2} AE \left(\frac{\partial u}{dx}\right)^2 \mathrm{d}x - \int_0^L up\mathrm{d}x$

Assume p is a constant term with value p_0. For this given problem the interpolation functions are as follows :

A. $u^h(x) = u_1\left(1 - \frac{x}{L}\right) + u_2\left(\frac{x}{L}\right)$

B. $N^h(x) = \alpha$ where α is constant.

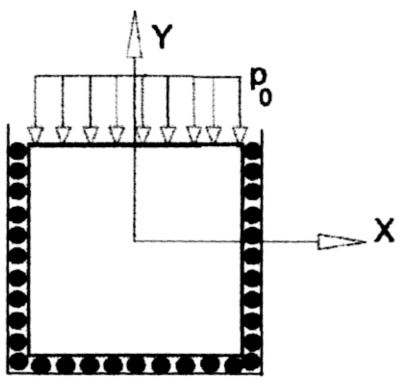
a) Using interpolation functions A and B calculate the discrete equations using functional 1.

b) Repeat the same procedure using interpolation function A and functional 2.

c) Eliminate and compare two discrete sets of equations.

11\. a) What is 'incremental plastic strain theory' ? 5

b) A piece of metal is compressed in a rigid die, as schematically shown in the figure. Assuming that the material is free to expand in the z-direction, find the pressure p_0 at the start of plastic deformation. The yield stress is σ_Y and Poisson's ratio is ν. The material is of von Mises type. 10



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