Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / M.Tech-IT(SE) / SEM-1 / MSE-104 / 2009-10

2009

DISCRETE STRUCTURE

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. $5 \times 14 = 70$

1. a) Let U denote citizens of West Bengal ; f, g and h are relations defined on U by the following statements :

   i) $f(x)$ is the mother of x

   ii) $g(x)$ is a daughter of x

   iii) $h(x)$ is a wife of x .

   Which of the statements define correctly a function from U to U ? Give reasons. 8

   b) Let A = $\{1, 2, 3, 4, 5\}$ and $f : A \varnothing A$ defined by $f(1) = 2, f(2) = 2, f(3) = 4, f(4) = 5$ and $f(5) = 4$. Show that $ff = f$ and find a function $g : A \varnothing A$ such that $gf = f$ and $fg = f$. 6

2. a) Define a semi-group ; give an example. Show that if G be a semi-group containing the right identity and every $x \varepsilon G$ has its right inverse in G then G is a group. 8

b) Show that symmetries of the letter 'H' form a finite group with respect to a suitable composition. 6

3. a) When is a subgroup H of a group G called normal in G ? If H be normal in G, show that $Ha = aH$ for every $a \varepsilon G$. 6

b) Let $Z(G) = \{ x \varepsilon G \mid xa = ax \text{ for all } a \varepsilon G \}$.

Show that

i) $Z(G)$ is a subgroup of G

ii) $Z(G)$ is normal in G. 5 + 3

4. a) Define prime ideal in a ring R with the unit element ; prove that in the ring of integers $\mathbb{Z}$ an ideal generated by a prime p, (p), is a prime ideal. 6

b) Let R be a commutative ring ; prove that any ideal $I \prod R$ is a prime ideal if and only if R/I is an integral domain.

8

5. a) In a Boolean Algebra B, if $x / y = 1$ and $x \square y = 0$ hold then $y = \bar{x}$ – the complement of x, $x, y \varepsilon B$. 7

b) State principle of duality and show that

$$\overline{(x \,\square\, y)} = \overline{x} \,/\, \overline{y} \,.$$ 5 + 2

6. a) Define homomorphism between two groups G and G' ; if $\sigma : G \varnothing G'$ be a homomorphism, prove that

i) $\sigma(e) = e'$, e and e' being the identity element in G and G' respectively.

ii) $\sigma(a^{-1}) = (\sigma(a))^{-1}$ for every $a \, \varepsilon \, G$.

iii) if H be a subgroup of G then so also is $\sigma(H)$. 8

b) Let S_n , $n \geq 2$ be the symmetric group of degree n ; prove that exactly half of the permutations in S_n is even and the other half odd. 6

7. a) Define and illustrate a connected graph ; let G be a graph with p vertices and δ satisfies the inequality $\delta \geq \dfrac{p-1}{2}$ where δ is the minimum of the degrees of vertices of G. Prove that G is a connected graph. 2 + 4

b) Define edge and vertex connectivity of a connected graph. Prove that the vertex connectivity of a graph G cannot exceed the edge connectivity. 2 + 6

8. a) Find the generating function of the Fibonacci numbers a_n . 7

b) Show that $a_n^2 - a_{n-1}\, a_{n+1} = (-)^n$, where a_n is same as in Q. 8 (a). 7