



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech-IT(SE)/SEM-1/MSE-104/2009-10**

**2009**

**DISCRETE STRUCTURE**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any five questions.

5 × 14 = 70

1. a) Let  $U$  denote citizens of West Bengal ;  $f$ ,  $g$  and  $h$  are relations defined on  $U$  by the following statements :

i)  $f(x)$  is the mother of  $x$

ii)  $g(x)$  is a daughter of  $x$

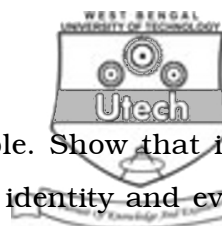
iii)  $h(x)$  is a wife of  $x$ .

Which of the statements define correctly a function from  $U$  to  $U$  ? Give reasons. 8

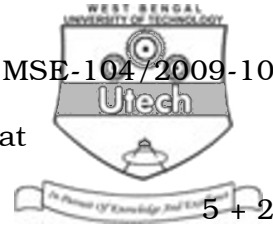
- b) Let  $A = \{ 1, 2, 3, 4, 5 \}$  and  $f : A \rightarrow A$  defined by

$f(1) = 2, f(2) = 2, f(3) = 4, f(4) = 5$  and

$f(5) = 4$ . Show that  $ff = f$  and find a function  $g : A \rightarrow A$  such that  $gf = f$  and  $fg = f$ . 6



2. a) Define a semi-group ; give an example. Show that if  $G$  be a semi-group containing the right identity and every  $x \in G$  has its right inverse in  $G$  then  $G$  is a group. 8
- b) Show that symmetries of the letter 'H' form a finite group with respect to a suitable composition. 6
3. a) When is a subgroup  $H$  of a group  $G$  called normal in  $G$  ? If  $H$  be normal in  $G$ , show that  $Ha = aH$  for every  $a \in G$ . 6
- b) Let  $Z ( G ) = \{ x \in G \mid xa = ax \text{ for all } a \in G \}$  .  
Show that  
i)  $Z ( G )$  is a subgroup of  $G$   
ii)  $Z ( G )$  is normal in  $G$ . 5 + 3
4. a) Define prime ideal in a ring  $R$  with the unit element ; prove that in the ring of integers  $\mathbb{Z}$  an ideal generated by a prime  $p$ ,  $( p )$ , is a prime ideal. 6
- b) Let  $R$  be a commutative ring ; prove that any ideal  $I \subseteq R$  is a prime ideal if and only if  $R/I$  is an integral domain. 8
5. a) In a Boolean Algebra  $B$ , if  $x/y = 1$  and  $x \sqcap y = 0$  hold then  $y = \bar{x}$  – the complement of  $x$ ,  $x, y \in B$ . 7



- b) State principle of duality and show that

$$\overline{(x \sqcup y)} = \bar{x} \wedge \bar{y}.$$

5 + 2

6. a) Define homomorphism between two groups  $G$  and  $G'$  ;  
if  $\sigma : G \rightarrow G'$  be a homomorphism, prove that
- $\sigma(e) = e'$  ,  $e$  and  $e'$  being the identity element in  $G$  and  $G'$  respectively.
  - $\sigma(a^{-1}) = (\sigma(a))^{-1}$  for every  $a \in G$ .
  - if  $H$  be a subgroup of  $G$  then so also is  $\sigma(H)$ . 8
- b) Let  $S_n$  ,  $n \geq 2$  be the symmetric group of degree  $n$  ; prove that exactly half of the permutations in  $S_n$  is even and the other half odd. 6
7. a) Define and illustrate a connected graph ; let  $G$  be a graph with  $p$  vertices and  $\delta$  satisfies the inequality  $\delta \geq \frac{p-1}{2}$  where  $\delta$  is the minimum of the degrees of vertices of  $G$ . Prove that  $G$  is a connected graph. 2 + 4
- b) Define edge and vertex connectivity of a connected graph. Prove that the vertex connectivity of a graph  $G$  cannot exceed the edge connectivity. 2 + 6
8. a) Find the generating function of the Fibonacci numbers  $a_n$  . 7
- b) Show that  $a_n^2 - a_{n-1} a_{n+1} = (-1)^n$  , where  $a_n$  is same as in Q. 8 ( a ). 7