#  <br> viesh <br> Name : <br> Roll No. <br> $\qquad$ romblan Invigilator's Signature : <br> $\qquad$ <br> CS/M.Tech (MBT, PHMC, PHMB)/SEM-1/MBT, PHMB, PHMC-104/2009-10 2009 <br> INTRODUCTORY MATHEMATICS 

Time Allotted : 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP - A
( Objective Type Questions )

1. Answer any ten of the following :
i) Find the derivative of $f(x)=\cos \left\{\ln \left(2 x^{2}+5\right)\right\}$.
ii) Find $\mathrm{d} y / \mathrm{d} x$ at $x=2$, if $y=x^{3}-5 x^{2}+4 x+2$.
iii) Choose the correct alternatives :

The curve of $f(x)=x \sin (1 / x)$ is symmetric with respect to
a) the $y$-axis
b) the $x$-axis
c) the origin
d) none of these.
iv) The equation $y=7 x /(x+5)$ represents a $\qquad$ . ( Fill in the blank )

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 singular.
vi) Matrix multiplication is a $\qquad$ operation. ( Fill in the blank )
vii) The equation $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}+\frac{\mathrm{d} y}{\mathrm{~d} x}-2 y=0$ is a $\qquad$ order and $\qquad$ degree differential equation. ( Fill in the blanks )
viii) Write down an example of a non-linear ordinary differential equation.
ix) Choose the correct alternatives :

The purpose of a measure of central tendency is to be describe what value of a distribution of scores?
a) The most typical/representative
b) The most surprising
c) The most significant
d) None of these.
x) The expression of the standard deviation as a percentage of the mean is the $\qquad$ ( Fill in the blank )

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xi) Choose the correct alternative :

Consider the set of numbers $11,12,13,14,15,16$ and 17. If we add 2 to each of these numbers, then the variance of the numbers
a) increases by 2
b) increases by 4
c) decreases by 2
d) remains unchanged.
xii) What is the domain of the function $f(x)=|x| / x$ ?

## GROUP - B <br> (Short Answer Type Questions )

Answer any three of the following. $3 \times 5=15$
2. Solve the equation : $\left(1+y^{2}\right) \mathrm{d} x+\left(1+x^{2}\right) \mathrm{d} y=0$.
3. If $y=a \log x+b x^{2}+x$ has its extreme values at $x=-1$ and $x=2$, determine the values of $a$ and $b$.
4. Find the eigenvalues of the matrix $A=\left(\begin{array}{ll}2 & 5 \\ 3 & 4\end{array}\right)$.
5. Find the median of the following data set :

| Income <br> (in Rs. ) | Below <br> 100 | $100-120$ | $120-140$ | $140-160$ | $160-180$ | $180-200$ | Above <br> 200 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 5 | 12 | 30 | 20 | 16 | 10 | 7 |

6. Sketch the function : $y=(x+1) /(x-2)$, showing the asymptotes, if any.
7. a) Semelparous organisms breed only once during their lifetime. Examples of this type of reproduction strategy can be found with Pacific salmon and bamboo. The per capita rate of increase $r$, can be thought of as a measure of reproductive fitness. The greater $r$, the more offspring an individual produces. The intrinsic rate of increase is typically a function of age $x$. Models for age-structured populations of semelparous organisms predict that the intrinsic rate of increase as a function of $x$ is given by

$$
r(x)=\ln [l(x) m(x)] / x
$$

where $l(x)$ is the probability of surviving to age $x$ and $m(x)$ is the number of female births at age $x$. Suppose that $l(x)=e^{-a x}, m(x)=b x^{c}$ where $a, b, c$ are positive constants. Find the optimal age of reproduction.
b) Find the maximum and minimum values of the function $f(x)=x^{3}-3 x^{2}-6 x+1$. $8+7$

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8. a) Find the critical point of the function

$$
f(x, y)=160 x-3 x^{2}-2 x y-2 y^{2}+120 y-18
$$

State whether $f(x, y)$ is maximum or minimum at this critical point.

Suppose now that the constraint $x+y=35$ is imposed. Use the method of Lagrange multiplier to determine the new critical point of the function $f(x, y)$ subject to the constraint mentioned above.
b) A student has a part-time job for which he is paid $\$ 8 /$ hour. His utility function for earning $\$ I$ and spending $S$ hours studying is given by
$u(I, S)=I^{1 / 4} S^{3 / 4}$.

Total amount of time that the student spends each week working and studying is 100 hours. How should he divide the time to maximize his utility?
9. a) Solve the equation : $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}+4 y=x^{2}$
b) Solve the equation : $\left(1-x^{2}\right) \frac{\mathrm{d} y}{\mathrm{~d} x}-x y=1$
c) A catalyst for a chemical reaction is a sybstance that controls the rate of the chemical reaction without changing the catalyst itself. An autocatalytic reaction is one whose product is a catalyst for its own formation.
$A+X \xrightarrow{k} X$
The rate of this reaction $v=\mathrm{d} x / \mathrm{d} t$ is given by the formula $v=k x(a-x)$, where $a$ is the initial concentration of the substance $A, x$ is the concentration of the product $X$ and $k$ is the rate constant of the reaction. Find the concentration $x$ that produces the maximum rate of reaction. $6+6+3$
10. a) A manufacturer of flashlight batteries took a sample of 13 batteries from a day's production and used them continuously until they were drained. The number of hours they were used until failure are given below :

342, 426, 317, 545, 264, 451, 1049, 631, 512, 266, 492, 562, 298
i) Compute the mean, median and mode. Looking at the distribution of times to failure, which measures of location do you think are most appropriate and which least appropriate to use for these data? Why?
ii) Calculate the range and standard deviation.
iii) What would you advise if the manufacturer wanted to be able to say in advertisements that these batteries "should last 400 hours" ?

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b) The operations manager of a package delikery service is deciding on whether to purchase a new fleet of trueks. When packages are stored in the trucks in preparation for delivery, two major constraints need to be considered - the weight ( in pounds ) and the volume ( in cubic feet ) for each item. The operations manager samples 200 packages, and finds that the mean weight is $26 \cdot 0$ pounds, with a standard deviation of 3.9 pounds, while the mean volume is 8.8 cubic feet with a standard deviation of $2 \cdot 2$ cubic feet. How can the operations manager compare the variation of the weight and the volume ?

$$
(4+5+2)+4
$$

11. a) Find the eigenvectors of the matrix $A=\left(\begin{array}{ll}1 & 1 \\ 0 & 1\end{array}\right)$.
b) During a laboratory experiment, muscular contractions of a frog muscle were measured against different doses of a given drug. The extent of contraction was considered as the response to the drug. The observations are given below :

| Serial no. of experiment | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Dose of drug | $0 \cdot 3$ | $0 \cdot 4$ | $0 \cdot 6$ | $0 \cdot 8$ | $0 \cdot 9$ |
| Response to drug | $54 \cdot 0$ | $59 \cdot 0$ | $60 \cdot 0$ | $65 \cdot 0$ | $70 \cdot 0$ |

For the above data, calculate the correlation coefficient and discuss about its significance.
c) If $f(x)=3 x^{2}+x, g(x)=x+4$, determine $f g(x)$ and $g f(x)$.

$$
4+8+3
$$

