Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (ME/MSS/SE/MTI)/SEM-1/MSS-101/ME-101/SE(CE)-101/MM(ME)-101/2012-13

2012
ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions.   5 × 14 = 70

1. a) The probabilities that *X*, *Y* and *Z* becoming the Principal of a certain college are respectively 0·3, 0·5 and 0·2. The probabilities that "Student Aid Fund" will be introduced in the college, if *X*, *Y* and *Z* become Principal are 0·4, 0·6 and 0·1 respectively. Given that "Student Aid Fund" has been introduced. Find the probability that Y has been appointed as Principal.

    b) Suppose 8% of the inhabitants of Kolkata are cricket fans. Determine approximately the probability that 10 inhabitants chosen at random include at least 2 cricket fans. How many among 500 samples of 10 inhabitants each will contain at least 2 cricket fans ?

2. a) In a certain car factory turning razor blades, there is a small chance, $\frac{1}{500}$ for any blade to be defective. The blades are in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

   b) The distribution of marks received in an examination is normal, 44% candidates got marks below 61, 4% candidates got marks above 80. Find mean and s.d. of the distribution. What is percentage rate of number of candidates receiving above 65 marks ?

3. a) If the random variable X follows a normal distribution whose mean is 18 and s.d. 25, find the value of $P(-31 < X < 67)$ and $P(x < 67 \ / \ x > 18)$.

   b) The length of life X of certain computers is approximately normally distributed with mean 800 hrs, and s.d. 40 hrs. If a random sample of 30 computers has an average life of 788 hrs., test the null hypothesis that $\mu = 800$ hrs. against alternative that $\mu = 800$ hrs at 0·5 % level of significance.

4. a) Find eigenvalues and normalized eigenvectors of the matrix
   $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

   b) Prove that product of the eigenvalues of a square matrix in equal to its determinant.
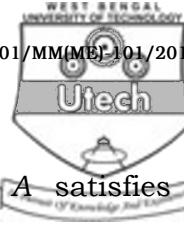
5. a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then verify that A satisfies its

   characteristic equation.

   b) Find a matrix P which transforms the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to a diagonal form.

6. Solve any *two* of the following :

   i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = x^3$.

   ii) $2\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + y = 0$.

   iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = e^{-x}$.

   iv) $\dfrac{d^2y}{dx^2} = \dfrac{a}{y^2}$.

7. a) Obtain D'Alembert's solution of the wave equation
   $$C^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

   b) Prove that the function $\dfrac{\cos\theta}{\gamma^2}$ satisfies the

   Laplace's equation in spherical polar coordinate system.

8. a) Solve the equation $x^4 - x - 10 = 0$ by Newton-Raphson

   method near $x = 0$, correct up to 4 diecimal places.

   b) Solve the equation by Rayleigh-Ritz method :

   $\dfrac{d^2y}{dx^2} + y + x = 0$, $0 \le x \le 1$, $y(0) = y(1) = 0$.

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