

# CS/M.Tech (MTI)/SEM-1/MTI-105/2009-10 2009 <br> ADVANCED ENGINEERING MATHEMATICS 

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions. $5 \times 14=70$

1. a) Using Newton-Raphson method, calculate the approximate value of the root of the equation $\cos 2 x=x$.
b) Using Cayley-Hamilton theorem, find the matrix from the following matrix :
$A=\left[\begin{array}{rr}\cos \theta & -\sin \theta \\ \sin \theta & \cos \theta\end{array}\right]$
2. a) Find the eigenvalues and eigenvectors of the following matrix :

$$
A=\left[\begin{array}{rrr}
1 & 1 & 3 \\
1 & 3 & -3 \\
-2 & -4 & -4
\end{array}\right]
$$

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3. a) From the following table, find the value of $y$ at $x=0.5$ :

| $x$ | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $54 \cdot 6$ | $-10 \cdot 7$ | $-1 \cdot 8$ | $2 \cdot 1$ | $31 \cdot 0$ |

b) If $A=\left[\begin{array}{lll}1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1\end{array}\right]$, show that $A^{2}-4 A-5 I=0$. Hence find $A^{-1}$.
4. a) Using Lagrange's interpolation formula, find the value of $y$ at $x=5$ from the following table :

| $x$ | 0 | 1 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 1 | -1 | 1 | -1 |

b) If $A=\left[\begin{array}{rr}2 & -1 \\ 0 & 1\end{array}\right]$ and $B=\left[\begin{array}{rr}1 & 0 \\ -1 & -1\end{array}\right]$, then show that

$$
(A+B)^{2} \neq A^{2}+2 A B+B^{2} .
$$

5. a) Integrate $\int_{1}^{5} \sqrt{126-x^{3}} \mathrm{~d} x$ by using Simpson's one-third rule, taking 4 intervals.
b) Find the polynomial which yields the table given below :

| $x$ | 2 | 4 | 6 | $0 \times 10$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -1 | 5 | 19 | 41 | 71 |

6. a) Find the solution of the following system of equations by Gauss-Seidel method correct upto 2 plases of decimal :
$80 x+11 y-5 z=90$
$5 x+50 y-12 z=100$
$3 x+5 y+30 z=70$
b) A book of 620 pages has 490 type errors. If the errors are randomly distributed throughout the book, what is the probability that
i) a particular page will have no more that two errors
ii) each of 10 given pages will be free of errors?
7. a) In a bolt factory, machines $A, B, C$ manufacture $25 \%$, $35 \%$ and $40 \%$ of the total output respectively. Of the outputs $5 \%, 4 \%$ and $2 \%$ are defective bolts. $A$ bolt is drawn at random from the total product mixed up and is found to be defective. What is the probability that it was produced by $A$ ?
b) In course of an experiment, 15 mosquitoes wese pyt in each of 120 jars and were next subjectê to a dose of DDT. After 4 hours, the numbers alive in each jar were counted and the following frequency distribution was obtained :

| No. of mosquitoes <br> alive $(x)$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of jars $(f)$ | 2 | 12 | 14 | 22 | 28 | 17 | 13 | 10 | 2 | 120 |

Fit a binomial distribution to the frequency distribution of number of alive mosquitoes per jar of 15 mosquitoes for 120 jars.
8. a) If the distribution of marks received in an examination is normal, $44 \%$ candidates got marks below 61, 4\% candidates got marks above 80. Find mean and standard deviation of distribution.
b) Calculate the mean and standard deviation of Poisson Distribution and show that mean is equal to variance.

