Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech(MMS)/SEM-1/MMS-106/2009-10

2009

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. a) Prove that exactly one of the events A and B occurs, whose probability is $P(A) + P(B) - 2P(A \cap B)$.

   b) If $P(A) = \frac{1}{2}$, $P\left(\bar{B}\right) = \frac{2}{3}$, $P\left(\bar{A}\bar{B}\right) = \frac{5}{6}$, are the events A and B independent ?

2. a) Two newspapers X and Y are published in a certain city. It is estimated from a survey that 16% read X, 14% read Y and 5% read both the newspapers. Find the probabilities that a randomly selected person

      i) does not read any newspaper and

      ii) reads only Y.

   b) For what value of a,

      $f(x) = ax, \quad x = 1(1)20,$

      $f(x) = 0, \quad$ otherwise

   will be the *p.m.f.* of random variable X. Find $P(X > 0 \,/\, X < 2)$. Find the expectation of X.

3. a) The probabilities of *X*, *Y* and *Z* becoming the principal of a college are 0, 3, 0·5 and 0·2 respectively. The probabilities that "Student Aid-Fund" will be introduced in the college, if *X*, *Y* and *Z* become principal are 0·4, 0·6 and 0·1 respectively. Given that "Student Aid-Fund" has been introduced. Find the probability that *Y* has been appointed as a principal.

   b) Let us suppose that 8% of inhabitants of Kolkata are cricket fans. (i) Determine approximately that 10 inhabitants chosen at random include at least 2 cricket fans. (ii) How many among 500 samples of 10 inhabitants each will contain at least 2 cricket fans ?

4. a) A book of 620 pages has 490 type mistakes. If these errors are randomly distributed throughout the book, (i) what is the probability that a given page will have no more than two errors and (ii) what is the probability that each of 10 given pages are free of errors ?

   b) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective.

5. a) If the weekly wages of 10,000 workers in a factory follows normal distribution with mean and s.d. Rs. 70 and Rs. 5 respectively, find expected number of workers whose weekly wages are (i) between Rs. 66 and Rs. 72 and (ii) less than Rs. 66. Given that $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0\cdot4} e^{-t^2/2}\, dt =$ 0·6554 and $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0\cdot8} e^{-t^2/2}\, dt = 0\cdot7881.$

b) The length of life X of certain computers is approximately normally distributed with mean 800 hrs and s.d. 40 hrs. If the random sample of 30 computers has an average of 788 hrs., test the null hypothesis that μ = 800 hrs. against the alternative hypothesis $\mu \neq 800$ at 5% level of significance.

[Given that $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-t^2/2}\, dt = \Phi(z_1)$, then z_1 = 1·96 at 5% level]

6. a) Using Cayley-Hamilton's theorem, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

b) Diagonalize the matrix given below :

$$\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$$

7. a) Using Newton-Raphson method, calculate approximately the real root of the equation $x^3 + 3x^2 - 10 = 0$ up to two places of decimal.

b) Use Simpson's rule to evaluate the integral $\int_1^5 \sqrt{x^3 - 1}\, dx$, taking 4 equal intervals upto 4 places of decimal.

8. a) Show that the sum of the eigenvalues of a matrix is the sum of the elements of the principal diagonal.

b) Using Gauss-Seidel iteration method, solve the following system of equations correct up to two places of decimal :

$$10x - 4y + z = 23$$

$$x + 10y + 3z = 17$$

$$-2x + 5y + 20z = 14.$$