Name : ..............................................................

Roll No. : ..............................................................

Invigilator's Signature : ..............................................

CS/M.TECH (ME)/SEM-1/M-1101/2009-10

# 2009

# ADVANCED OPTIMIZATION STATISTICAL METHODS

*Time Allotted* : 3 Hours Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

1. Answer any *five* questions : 5 × 5 = 25

   a) What are multiple and partial correlations (3 variables) ?

   b) Write short note on Geometric Programming tecnique.

   c) Define Spearman rank correlation and correlation coefficient.

   d) Write short note on Stomachic programming.

   e) This line of regression of $y$ on $x$ on $y$ are respectively $y = x + 5$ and $16x = 9y - 94$. Find the variance of $x$ if the variance of $y$ is 16. Also find the coveriance of $x$ and $y$.

f) If $x_1$ , $x_2$ , $x_3$ are three variates measured from their means with $N = 10$

$\sum x_1^2 = 90$, $\sum x_2^2 = 160$, $\sum x_3^2 = 40$,

$\sum x_1 x_2 = 60$, $\sum x_2 x_3 = 60$, $\sum x_3 x_1 = 40$.

Calculate the partial correlation coefficient $r_{31 \cdot 2}$ and multiple correlations coefficient $R_{1 \cdot 23}$

g) Define the test for goodness of fit using Chi-square ( $\chi^2$ ) distribution.

## GROUP – B

Answer any *three* questions. $3 \times 15 = 45$

2. a) Determine the optimal solution for the following NLPP and check it maximizes or minimizes the objective function :

$$\text{Optimize } z = 2x_1 - x_1^2 + 2x_2$$

$$\begin{aligned} \text{subject to} \quad & 2x_1 + 3x_2 \le 6 \\ & 2x_1 + x_2 \le 4 \\ & x_1 , x_2 \ge 0. \end{aligned}$$

b) If the equations of two Regression lines obtained in a correlation analysis are $3x + 12y - 19 = 0$ and

$9x + 3y = 46$. Determine which one is regression equation of $y$ on $x$ and which one is the regression equation of $x$ on $y$. Find the mean of $x$ and $y$, Correlation coefficient and ratio of Standard Deviation of $x$ and $y$. 8 + 7

3. a) Solve the following NLPP by Lagrange Multipliers method :

Minimize $z = x_1^2 + x_2^2 + x_3^2$

subject to $4x_1 + 2x_2^2 + 2x_3 = 14$

$x_1, x_2, x_3 \geq 0$.

b) The followiing figures give the number of defectives in 20 samples each containing 2000 items :

423, 430, 216, 341, 223, 322, 280, 306, 337, 305, 356, 402, 216, 264, 126, 409, 193, 326, 280, 390.

Calculate the values for central line and the control limits for $p$ chart ( fraction-defective chart ). 8 + 7

4. a) In a sample of 8 observations, the sum of squared deviations of items from the mean was 94·5. In another sample of 10 observations the value was found to be 101·7. Test whether the difference is significant at 5% level. ( You are given that 5% level, critical value of $F$ for $\upsilon_1 = 7$ and $\upsilon_2 = 9$ degrees of freedom is 3·29 and for $\upsilon_1 = 8$ and $\upsilon_2 = 10$ d.f. its value is 3·07.

b) Find the optimum integer solution to the following integer programming problem :

Maximize $z = x_1 + x_2$

Subject to the constraints $3x_1 + 2x_2 \leq 5$

$x_2 \leq 2$

Where $x_1, x_2$ are integer's $\geq 0$. 7 + 8

5. a) Find the minimum value of $z = y_1^2 + y_2^2 + \ldots + y_n^2$

subject to $y_1 + y_2 + \ldots + y_3 = c\ (\neq 0)$

$y_1 \geq 0,\ j = 1, 2, \ldots, n.$

b) Solve the following NLPP using Kuhn-Tucker condition :

Maximize $z = 2x_1^2 - 7x_2^2 + 12x_1 x_2$

subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0.$ 8 + 7