Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(ME)/SEM-1/ME-101/2009-10

2009

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions.   5 × 14 = 70

1. Let $f(x)$ be a periodic function with period 2π $(-\pi < x < \pi)$. Let $f(x)$ be expressed as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Then prove that

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx, \quad n = 1, 2, \ldots..$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx, \quad n = 1, 2, \ldots..$$

2. Find the Fourier expansion of $f(x) = x^2$ in $-\pi \le x \le \pi$, and deduce that

a) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$

b) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots = \frac{\pi^2}{12}$

3. Prove that $f(x) = x$ can be expanded in a series of cosines in $0 < x < \pi$, as

$$x = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots\right]$$

Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$

4. Find Fourier transform of

$f(x) = 1$, $|x| \le a$

$= 0$, otherwise,

where a is a positive real number. Hence deduce that

$$\int_0^\infty \frac{\sin t}{t}\, dt = \frac{\pi}{2}.$$

5. Find the Fourier of the function $f(x)$ defined by

$f(x) = 1 - x^2$, $|x| < 1$

$= 0$, $|x| \ge 1$.

Hence prove that

$$\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}.$$

6. Find the Laplace transform of

a) $\dfrac{1-\cos 2t}{t}$

b) $\dfrac{e^{-3t}\sin 2t}{t}$.

7. a) Define singular point of a differential equation. Solve in series the equation $\dfrac{d^2y}{dx^2} - y = 0$.

b) Show that $\dfrac{d}{dx}\left\{ x^n J_n(x) \right\} = x^n = J_{n-1}(x)$.

8. a) Express $x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

b) Establish D'Alembert's general solution of the wave equation in the form

$$u = f_1(x + ct) + f_2(x - ct) \ .$$