

Name :

Roll No. :

Invigilator's Signature :

**CS/M.Tech(ME)/SEM-1/ME-101/2009-10
2009**

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer any *five* questions.

5 × 14 = 70

1. Let $f(x)$ be a periodic function with period 2π ($-\pi < x < \pi$).
Let $f(x)$ be expressed as

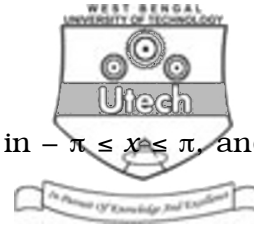
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Then prove that

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, n = 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx, n = 1, 2, \dots$$



2. Find the Fourier expansion of $f(x) = x^2$ in $-\pi \leq x \leq \pi$, and deduce that

$$\text{a) } \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$\text{b) } \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

3. Prove that $f(x) = x$ can be expanded in a series of cosines in $0 < x < \pi$, as

$$x = \frac{\pi}{2} - \frac{4}{\pi} \left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right]$$

Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

4. Find Fourier transform of

$$f(x) = 1, |x| \leq a$$

$$= 0, \text{ otherwise,}$$

where a is a positive real number. Hence deduce that

$$\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}.$$

5. Find the Fourier of the function $f(x)$ defined by

$$f(x) = 1 - x^2, |x| < 1$$

$$= 0, |x| \geq 1.$$

Hence prove that

$$\int_0^{\infty} \frac{\sin s - s \cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}.$$



6. Find the Laplace transform of

a) $\frac{1 - \cos 2t}{t}$

b) $\frac{e^{-3t} \sin 2t}{t}$.

7. a) Define singular point of a differential equation. Solve in series the equation $\frac{d^2y}{dx^2} - y = 0$.

b) Show that $\frac{d}{dx} \{ x^n J_n(x) \} = x^n = J_{n-1}(x)$.

8. a) Express $x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

b) Establish D'Alembert's general solution of the wave equation in the form

$$u = f_1(x + ct) + f_2(x - ct) .$$

