Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(ME-O)/SEM-1/MM(ME)-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Attempt any *five* questions. $5 \times 14 = 70$

1. a) Given that the mode of the following frequency distribution of 70 students is 58·75. Find the missing frequencies f_1 and f_2.

Class interval	52 — 55	55 — 58	58 — 61	61 — 64
Frequency	15	f_1	25	f_2

b) Determine the constants *a* and *b* by the method of least squares such that $y = ae^{bx}$ fits the following data :

x	2	4	6	8	10
y	4·077	11·084	30·128	81·897	222·62

7 + 7

2. a) If *r* be the correlation coefficient for a set of bivariate data, prove that $-1 \le r \le 1$. Discuss the cases $r = \pm 1$.

b) For two variables *x* and *y*, the two regression lines are $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$. Identify which one is of *y* on *x*. Find means of *x* and *y*. Find the correlation coefficient between *x* and *y*. Estimate the value of *x* when $y = 1{\cdot}5$. 7 + 7

3. a) Obtain an estimate of error in polynomial interpolation.

b) Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x	0	1	3	4
y	– 12	0	12	24

7 + 7

4. a) Solve the following System by Gauss-Seidel method corrected up to two decimal places :

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

b) Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$ correct to two decimal places using power method.

5. a) Discuss how the nodes (x_k) and the weights (λ_k) are determined in Gauss-Legendre integration formula

$$\int_{-1}^{1} f(x)\,dx = \sum_{0}^{n} \lambda_k f(x_k).$$

b) Solve the following system of equations, correct to 2 decimal places, by Newtow-Raphson method with (1, 2) as initial approximation :

$$x + y = 3x^2,\ y^3 - 2 = 4x^3.$$ 7 + 7

6. Solve the BVP :

$y'' + 2y = x, \; 0 < x < 1$

$y(0) = 0, \; y(1) = 0.$

by Rayleigh-Ritz method using the approximating function $w(x) = x(1-x)\left(a_1 + a_2 x\right)$. 14

7. a) State Fourier intergal theorem.

b) Find the Fourier sine transform of $f(x) = \dfrac{1}{xe^x}$

c) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the initial temperature is

$$u(x, 0) = x, \quad 0 \le x \le 50$$
$$= 100 - x, \quad 50 \le x \le 100.$$

Find the temperature $u(x, t)$ at any time t by the method of separation of variables. 2 + 5 + 7

8. a) A string is stretched and fixed between two points $x = 0$ and $x = L$. Motion is initiated by displacing the string in the form

$$u = a \sin \frac{\pi x}{L}$$

and released from rest at $t = 0$. Find the displacement of any point on the string at any time t by using integral transform technique.

b) Find the steady state temperature distribution in a large rectangular plate, the flat surfaces of which are insulated, when the temperature is prescribed by $f(x)$ along one edge of the plate and tends to zero along each of the other edges. 7 + 7