



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.TECH (MTT & MCP)/SEM-3/CS-312/2010-11**

**2010-11**

**NUMERICAL METHODS & PROGRAMMING**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any five questions.

1. a) Derive  $f[x_0, x_1, \dots, x_n] = \frac{1}{n! h^n} \Delta^n f_0$

and  $f[x_0, x_1, \dots, x_n] = \frac{1}{n! h^n} \nabla^n f_n$

where  $\Delta, \nabla$  forward and backward difference operator.  
 $x_0, x_1, \dots, x_n$  be equally spaced points i.e.

$x_i = x_0 + ih$  where  $i = 0, 1, \dots, n$ .

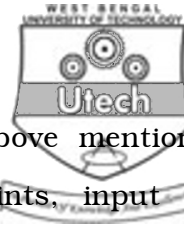
$f[x_0, x_1, \dots, x_n] \propto n^{\text{th}}$  Newton divided differences. 4

- b) Calculate the differences and obtain the forward difference polynomial from the following data :

$x :$	0.1	0.2	0.3	0.4	0.5
$f(x) :$	1.40	1.56	1.76	2.00	2.28

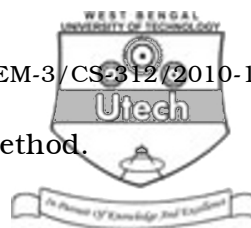
Interpolate at  $x = 0.25$  and  $x = 0.35$ .

5



- c) Write programme code to read the above mentioned data ( 5 points ) *i.e.* number of points, input the abscissas, and printing the same to write file, input of ordinates and printing the same. 5
2. a) Explain the principle involved in Runge-Kutta 4th order method. 5
- b) Solve the initial value problem :
- $$u' = -2tu^2 \quad u(0) = 1$$
- with  $h = 0.2$  on the interval  $[0, 0.4]$ . Use the fourth order classical Runge-Kutta method. Compare with the exact solution. 6
- c) Write programme code to compute  $f(t, u)$  as function subprogram, where  $f(t, u) = -2tu^2$  as in above. 3
3. a) Calculate the value of integral  $\int_0^2 \frac{dx}{5+3x}$  using  $\frac{3}{8}$  th Simpson's rule. 6
- b) Write a program to evaluate the above integral of  $f(x)$  using Simpson's rule of integration based on  $2n$  sub-intervals or  $2n + 1$  node points.

The values of  $a$ ,  $b$  and  $n$  are to be read and the integrand is written as a function sub-program. 8



4. a) Describe the Gauss-Seidel iteration method. 4

- b) Solve the system of equation

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

Using the Gauss-Seidel method.

Take the initial approximation as  $[x]^{(0)} = 0$  and perform three iteration. 10

5. a) Apply Newton-Raphson's method to determine a root of the equation

$$f(x) = \cos x - xe^x = 0$$

such that  $|f(x^*)| < 10^{-8}$ , where  $x^*$  is the approximation to the root. Take the initial approximation as  $x_0 = 1$ . 6

- b) Find the rate of convergence of the Newton-Raphson method. 4

- c) Write the algorithm to solve the equation mentioned above. 4

6. a) Define floating and fixed point representation of real number. 4

- b) Define relative error and absolute error, round-off-error. 4

- c) Define significant digits. 2

- d) What is illconditioned system of equation? 4



7. a) Describe the method to express the partial differentiation :

$$\left( \frac{\partial f}{\partial x} \right)_{(x_i, y_i)} ; \left( \frac{\partial f}{\partial y} \right)_{(x_i, y_i)} ;$$

$$\left( \frac{\partial^2 f}{\partial x^2} \right)_{(x_i, y_i)} ; \left( \frac{\partial^2 f}{\partial y^2} \right)_{(x_i, y_i)} .$$

4

- b) Find the Jacobian matrix for the system of equations

$$f_1(x, y) = x^2 + y^2 - x = 0$$

$$f_2(x, y) = x^2 - y^2 - y = 0$$

at the point ( 1, 1 ) using the methods with  $h = k = 1$ ,  
where  $x_i = x_0 + ih$ ,  $y_j = y_0 + jk$ ,  $i, j = 1, 2 \dots$ .

10

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