Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (MTT & MCP)/SEM-3/CS-312/2010-11

2010-11

NUMERICAL METHODS & PROGRAMMING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions.

1. a) Derive $f[x_0, x_1 \ldots\ldots x_n] = \frac{1}{n!\,n^n} \Delta^n f_0$

and $f[x_0, x_1 \ldots\ldots x_n] = \frac{1}{n!\,n^n} \nabla^n f_n$

where Δ, ∇ forward and backward difference operator. $x_0, x_1 \ldots\ldots, x_n$ be equally spaced points *i.e.*

$x_i = x_o + ih$ where $i = 0, 1, \ldots, n$.

$f[x_0, x_1, \ldots, x_n] \varnothing\, n^{th}$ Newton divided differences. 4

b) Calculate the differences and obtain the forward difference polynomial from the following data :

x :	0·1	0·2	0·3	0·4	0·5
$f(x)$:	1·40	1·56	1·76	2·00	2·28

Interpolate at $x = 0·25$ and $x = 0·35$. 5

c) Write programme code to read the above mentioned data (5 points) *i.e.* number of points, input the abscissas, and printing the same to write file, input of ordinates and printing the same. 5

2. a) Explain the principle involved in Runge-Kutta 4th order method. 5

b) Solve the initial value problem :

$$u' = -2tu^2 \quad u(0) = 1$$

with $h = 0{\cdot}2$ on the interval $[0, 0{\cdot}4]$. Use the fourth order classical Runge-Kutta method. Compare with the exact solution. 6

c) Write programme code to compute $f(t, u)$ as function subprogram, where $f(t, u) = -2tu^2$ as in above. 3

3. a) Calculate the value of integral $\int_0^2 \frac{dx}{5+3x}$ using $\frac{3}{8}$ th Simpson's rule. 6

b) Write a program to evaluate the above integral of $f(x)$ using Simpson's rule of intigration based on $2n$ sub-intervals or $2n + 1$ nodel points.

The values of a, b and n are to be read and the integrand is written as a function sub-program. 8

4. a) Describe the Gauss-Seidel iteration method. 4

b) Solve the system of equation

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

Using the Gauss-Seidel method.

Take the initial approximation as $[\,x\,]^{(0)} = 0$ and perform three iteration. 10

5. a) Apply Netwon-Raphson's method to determine a root of the equation

$$f(x) = \cos x - xe^x = 0$$

such that $\left| f(x^*) \right| < 10^{-8}$, where x^* is the approximation to the root. Take the initial approximation as $x_0 = 1$. 6

b) Find the rate of convergence of the Newton-Raphson method. 4

c) Write the algorithm to solve the equation mentioned above. 4

6. a) Define floating and fixed point representation of real number. 4

b) Define relative error and absolute error, round-of-error.

4

c) Define significant digits. 2

d) What is illconditioned system of equation ? 4

7. a) Describe the method to express the partial differentiation :

$$(\partial f / \partial x)_{(x_i, y_i)} \; ; \; (\partial f / \partial y)_{(x_i, y_i)} \; ;$$

$$(\partial^2 f / \partial x^2)_{(x_i, y_i)} \; ; \; (\partial^2 f / \partial y^2)_{(x_i, y_i)} \; .$$

4

b) Find the Jacobian matrix for the system of equations

$$f_1(x, y) = x^2 + y^2 - x = 0$$

$$f_2(x, y) = x^2 - y^2 - y = 0$$

at the point (1, 1) using the methods with $h = k = 1$, where $x_i = x_0 + ih$, $y_j = y_0 + jk$, $i, j = 1, 2 \ldots$.

10