Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M.TECH (MC-VLSI)/SEM-1/PGMVD-101/2011-12

2011
ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

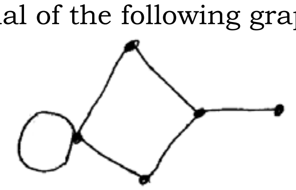
1. Answer any *five* questions : $5 \times 2 = 10$

   a) What do you mean by 'complement of a graph' ?

   b) What is the number of vertices in a 4-dimensional hypercube ? Draw a hypercube of dimension 3.

   c) Define a planar graph.

   d) Draw the dual of the following graph :



   e) Draw a digraph whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

f) Find the laplace transform of $f(t) = \cos \omega t$.

g) Determine the inverse Laplace transform of

$$F(S) = \frac{5}{3S-1}$$

h) For a function $f(x)$ of period 2L, write down the expression for the Fourier coefficient b_n.

i) Write down the Fourier transform of $f'(x)$ in terms of the Fourier transform of $f(x)$ and ω.

GROUP – B

Answer any *six* questions. $6 \times 10 = 60$

2. a) What do you mean by a clique ? Draw a connected graph with 7 vertices and 9 edges which contains a clique of size 4. 2 + 3

b) Prove that a tree with two or more vertices has at least two pendant vertices. 5

3. a) Prove that any tree with two or more vertices is 2-chromatic. 5

b) Define complete matching and maximal matching. State Hall's theorem in connection with matching. 5

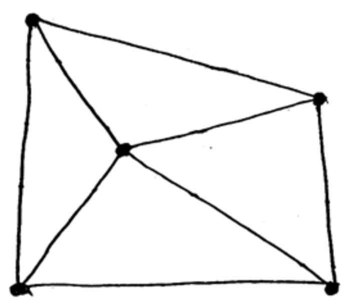
4. State and prove Euler's theorem for planar graphs. 10

5. Consider the following planar graphs G_1 and G_2 which are isomorphic :

a) Draw the dual G'_1 of G_1 and the dual G''_1 of G'_1. Examine whether G_1 and G''_1 are isomorphic. 6

b) Draw the dual G'_2 of G_2. Examine whether G'_1 and G'_2 are isomorphie. 4

6. a) Find the chromatic polynomial of the following graph :



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b) If there are 4 colours, in how many ways can the vertices of the graph in (a) be properly coloured ? 2

7. Using Laplace transform, solve the initial value problem,

$y'' + 4y = f(t),\ y(0) = y'(0) = 0$

in which $f(t) = \begin{cases} 0 & for \quad t < 3 \\ t & for \quad t \geq 3 \end{cases}$. 10

8. a) Using convolution property, solve the following integral equation :

$$y(t) = t + \int_0^t y(\tau) \sin(t-\tau)\, d\tau.$$ 6

b) Determine the Laplace transform of

$$f(t) = \frac{t}{2\beta} \sin \beta t.$$ 4

9. a) Find the Fourier transform of

$f(x) = e^{-ax^2}$ where $a > 0$. 7

b) Find the Z transform of $(\cos\theta + i\sin\theta)^n$. 3

10. a) Find the half-range Fourier sine series for the function $f(x) = x$ in the range $0 \le x \le 2$. Sketch the function within and outside of the given range. 6

b) Starting from the trigonometrie series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$

establish the representation of the complex Fourier series, *viz.* $f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$ 4