

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech(LT)/SEM-1 /MLT-101 /2012-13**

**2012**

**MATHEMATICAL AND COMPUTATIONAL METHODS**

*Time Allotted : 3 Hours*

*Full Marks : 70*

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

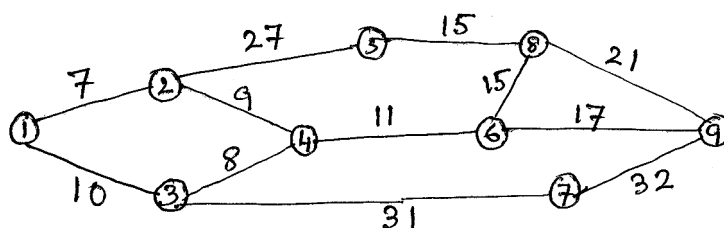
*Graph sheet will be supplied by the institution.*

*Attempt any seven questions.  $7 \times 10 = 70$*

1. IBM produces two kinds of memory chips (chip-I and chip-II) for memory usage. The unit-selling price is Rs. 1500 for chip - I and Rs. 2500 for chip-II . To make one chip-I , IBM has to invest 3 hours of skilled labour, 2 hours of unskilled labour and 1 unit of raw material. To make one chip-II, it takes 4 hours of skilled labour, 3 hours of unskilled labour and 2 units of raw material. The company has 120 hours of skilled labour, 60 hours of unskilled labour and 30 units of raw material available. IBM requires that at least 3 units of chip-II have to produce as per sale contract signed by IBM. Formulate the problem as an L.P.P. and solve it graphically.



2. Find the shortest path from node 1 to node 9 of the distance network shown in the following figure using Dijkstra's algorithm.



3. Reduce following game by dominance and find the value of the game :

|     | I | II | III | IV |
|-----|---|----|-----|----|
| I   | 3 | 2  | 4   | 0  |
| II  | 3 | 4  | 2   | 4  |
| III | 4 | 2  | 4   | 0  |
| IV  | 0 | 4  | 0   | 8  |

4. a) Solve by the method of separation of variables  $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$  where  $u(x, 0) = 6e^{-3x}$ .
- b) Use Picard's method to compute  $y(0.1)$  from the differential equation  $\frac{dy}{dx} = 1 + xy$ , given  $y = 1$  where  $x = 0$  correct up to 4 places of decimal.
5. Solve the one-dimensional wave equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ , given that  $y(0, t) = y(l, t) = 0 \forall t > 0$  and  $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0, \forall x$ . and  $y(x, 0) = \mu x(l - x), 0 \leq x \leq l$ .



6. Find the solution of the following one-dimensional heat equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  ( $0 < x < l$ ;  $t > 0$ ), subject to the boundary conditions :  $u(0, t) = u(l, t) = 0$ ,  $\forall t > 0$  and initial condition  $u(x, 0) = x$ ,  $l$  being the length of the bar.
7. Solve  $\frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 x}{\partial y^2} = 0$  which satisfies the conditions :  
 $u(0, y) = u(l, y) = u(x, 0) = 0$  and  $u(x, a) = \sin \frac{n\pi x}{l}$ .
8. a) Using third order Taylor's series expansion, compute  $y(1.1)$  from initial value problem  $\frac{dy}{dx} = xy$ ,  $y(1.0) = 2$ , correct up to 5 places of decimal.
- b) Given  $\frac{dy}{dx} = \frac{y-x}{y+x}$  with initial condition  $y(0)=1$ . Find  $y$  for  $x = 0.1$ , by Euler's method, correct up to four decimal places, taking step length  $h = 0.02$ .
9. Compute  $y(0.5)$  and  $y(1)$  by Runge-Kutta 4th order method from the equation  $\frac{dy}{dx} = \frac{1}{x+y}$  taking  $h = 0.5$  correct up to 4 places of decimal.
10. Compute  $y(0.4)$  by Milne's predictor corrector method from the equation  $\frac{dy}{dx} = xy + y^2$ , given that  $y(0) = 1$ ,  $y(0.1) = 1.1169$ ,  $y(0.2) = 1.2773$ ,  $y(0.3) = 1.5040$  correct up to 3 places of decimal.