Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (LT)/SEM-1/MLT-101/2011-12

2011

MATHEMATICAL AND COMPUTATIONAL METHODS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any seven questions. $7 \times 10 = 70$

1. On a metal shop two articles are produced : bucket lids and pressed metal handles. The lid takes 20 seconds to stamp, 30 seconds to form and 20 seconds to paint. The handle takes 40 seconds to stamp, 10 seconds to form and 16 seconds to paint. The profit margin on lids and handles are 6 paise and 9 paise respectively. The time available per day on each process in 8 hours and 20 seconds (30,000 seconds). How many lids and handles should be produced to maximize profit ? Find also the maximum profit.

2. Construct a network for the project, so that the following constraints are satisfied :

$A < D, E > A, B < F, D < F, C < G, H > C, F < I, I < G, E > A$

Time completion of each activity is given below :

Activity	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

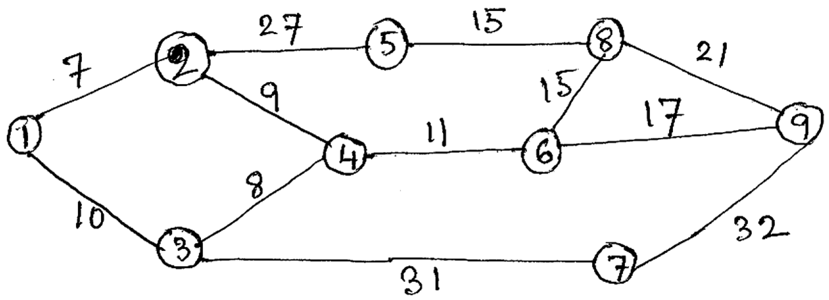
i) Draw the network and number the events.

ii) Find critical path.

3. Reduce the following game by dominance and find the value of the game :

	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

4. Find the shortest path from node 1 to node 9 of the distance network shown in the following figure using Dijkstra's algorithm.



5. a) Find, by using Picard's method, y (0·1) from the equation $\frac{dy}{dx} = x + x^2 y$; given that y (0) = 1.

   b) Using third order Taylor's series expansion, compute y (1·1) from the initial value problem $\frac{dy}{dx} = xy$, given that y (1) = 2.

6. a) Compute y (0·3) from $\frac{dy}{dx} = 1 + xy$ by modified Euler's method given that y (0) = 2, taking h = 0·3.

   b) Use the method of separation of variables technique to solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ with $n(0, y) = e^{-5y}$.

7. Using fourth order Runge-Kutta method solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ for x = 0·5 and x = 1 taking h = 0·5.

8. Compute y (0·4) by Milne's predictor corrector method from the equation $\frac{dy}{dx} = xy + y^2$, given that y (0) = 1, y (0·1) = 1·1169, y (0·2) = 1·2773, y (0·3) = 1·5040.

9. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, in a square of length π in the xy-plane satisfying the following boundary condition : u (0, y) = 0, u (π, y) = 0, u (x, π) = 0 and $u(x, 0) = \sin^2 x,\ 0 < x < \pi$.

10. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables subject to the boundary conditions : u (O, t) = 0, u (L, t) = 0 and initial condition $u(x, 0) = \frac{dx(L-x)}{L^2}$, $0 < x < L$. where d and L are constants.

11. A string of length L is stretched and fastended to two fixed points. Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ with the boundary condition y (O, t) = 0, y (L, t) = 0 and with the initial displacement

$$y(x, 0) = f(x) = \begin{cases} \dfrac{2kx}{L}, & \text{if } 0 < x < \dfrac{L}{2} \\[2ex] \dfrac{2k(L-x)}{L}, & \text{if } \dfrac{L}{2} < x < L \end{cases}$$