Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.TECH (IT-SE)/SEM-1/MSE-104/2011-12

2011
DISCRETE STRUCTURE

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. a) Prove that a non-trivial finite ring having no divisor of zero is a ring with unity. 4

   b) Define Integral Domain. 2

   c) A commutative ring R with unity is an integral domain iff for every non-zero element a in R $a.u = a.v \Rightarrow u = v$; where $u, v \in R$. Prove. 8

2. a) Let (G, •) be an Abelian group and H = { a^2 | a ∈ G }. Prove that H is a subgroup of G. 8

   b) Does the set of all 2 × 2 non-singular matrices over integers form a group under matrix multiplication ? Justify. 6

3. a) A mapping $f : R \to \mathbb{R}$ is defined by $f(x) = x^2 + x$, $x \in \mathbb{R}$. Show that f is neither injective nor surjective. 5

   b) Prove that the inverse of inverse of an element in a group (G, •) is the element itself. 4

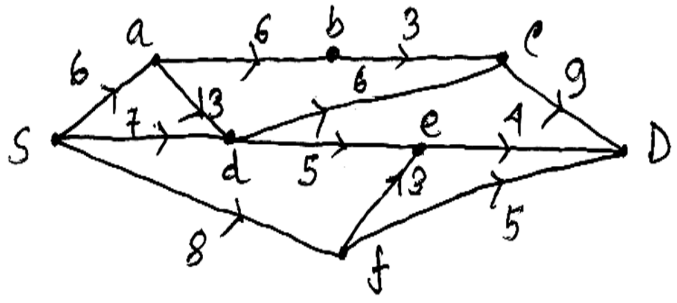
   c) Show that the following relation R on $\mathbb{Z}$ is an equivalence relation $R = \{ (a, b) ; a, b \in \mathbb{Z}$ and $a^2 + b^2$ is a multiple of 2 $\}$. 5

4. a) Prove that a connected graph is a tree if and only if it has fewer edges than vertices. 6

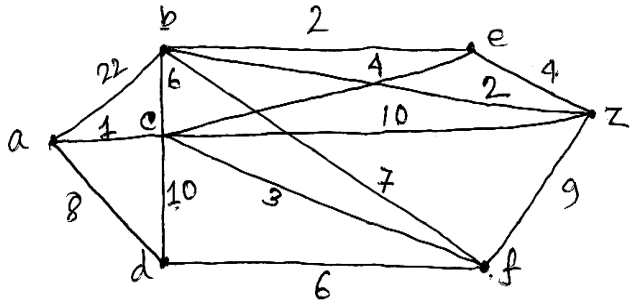
   b) Show that number of pendant vertices in a binary tree having n vertices is $(n + 1) / 2$. 4

   c) Define level of a vertex in a binary tree and illustrate through example. 4

5. Use Ford-Fulkerson algorithm to find a maximal flow for the following network :



6. Find a shortest path from the vertex a to the vertex z by using Dijkstra's algorithm :



7. Find a minimal spanning tree for the following graph by using Prim's algorithm :