



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech(EE)/SEM-1/MEE-1.5.4/2009-10  
2009**

**OPTIMAL CONTROL AND ESTIMATION**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer any *five* questions. 5 × 14 = 70

1. a) Derive Euler-Lagrange equation to determine a curve  $x(t)$  connecting two points  $(x_o, t_o)$  and  $(x_f, t_f)$  such that the integral along the curve of some given function  $F(x, \dot{x}, t)$  is a minimum. 7

- b) Find the extremal curves for the functional 7

$$J = \int_0^{t_f} \sqrt{\{1 + \dot{x}^2(t)\}} dt$$

2. a) State Pontryagin's maximum principle. Discuss the steps involved in solving optimal control problems using this principle. 7



- b) The dynamics of a system is described by

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = u(t)$$

This system is to be controlled, minimizing the PI,

$$J = (X, u) \frac{1}{2} \int_0^2 u^2(t) dt$$

Find a set of necessary conditions for the optimal control. 7

3. a) Using the definition, determine the differential of the functional,

$$J(X) = \int_{t_0}^{t_f} \left[ x_1^2(t) + x_1(t)x_2(t) + x_2^2(t)2\dot{x}_1(t)\dot{x}_2(t) \right] dt$$

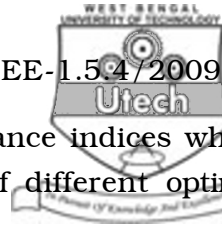
Assume that the end points are specified.

- b) Find the extremals for the functional,

$$J(X) = \int_0^{\pi/2} \left[ \dot{x}_1^2(t) + 2x_1(t)x_2(t) + \dot{x}_2^2(t) \right] dt$$

with  $x_1(0) = 0$ ,  $x_1(\pi/2) = 1$ ,  $x_2(0) = 0$ ,  $x_2(\pi/2) = 1$ .

- c) Let  $f(x) = -x_1x_2$  and  $g(x) = x_1^2 + x_2^2 - 1$ . What are the potential candidates for minima of  $f(x)$  subject to the condition  $g(x) = 0$ . 3 + 3 + 8



4. a) Briefly discuss the different performance indices which are generally used for formulation of different optimal control problems.

- b) The auto-correlation function of a noise signal is given by,

$R_x(\tau) = \sigma^2 \exp(-\beta|\tau|)$  where  $\sigma$  and  $\beta$  are two constants.

Obtain the transfer function of the shaping filter that will convert a unity white noise signal to this particular noise signal. 7 + 7

5. a) Distinguish between a random variable and a random process. What do you mean by strict-sense stationary, wide-sense stationary and ergodicity of a random process ?

- b) Show how the mean and covariance of the state vector propagate through time when a linear dynamic system is excited by a zero-mean white noise input. 6 + 8

6. a) What do you mean by prediction, filtering and smoothing problems ? In what sense is the Kalman filter an optimal filter ?

- b) A linear discrete-time system is described by,

$$x_k = x_{k-1} + w_{k-1}$$

$$z_k = x_k + v_k$$

where the process noise and the measurement noise are zero-mean white noises with intensities 1 and 2 respectively.

Calculate the Kalman gain  $K_k$  and the estimation error covariance  $P_k$  for  $k = 1$  and 2 assuming  $P_0 = 10$ . Also determine their steady state values.

CS/M.Tech(EE)/SEM-1/MEE-1.5.4/2009-10



- c) Explain how colored process disturbance situation can be accommodated in a standard Kalman filter setting.

4 + 6 + 4

7. a) Derive the return difference inequality property of the infinite-time LQR controller and show that this controller yields a minimum gain margin of infinity and phase margin of  $60^\circ$  for a SISO minimum phase system.
- b) Write a short note on Loop Transfer Recovery method.

9 + 5

---