Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech(EE)/SEM-1/MEE-1.5.4/2009-10

2009

OPTIMAL CONTROL AND ESTIMATION

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. a) Derive Euler-Lagrange equation to determine a curve $x(t)$ connecting two points (x_o, t_o) and (x_f, t_f) such that the integral along the curve of some given function $F(x, \dot{x}, t)$ is a minimum. 7

   b) Find the extremal curves for the functional 7

$$J = \int_0^{t_f} \sqrt{\{1 + \dot{x}^2(t)\}}\, dt$$

2. a) State Pontryagin's maximum principle. Discuss the steps involved in solving optimal control problems using this principle. 7

b) The dynamics of a system is described by

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = u(t)$$

This system is to be controlled, minimizing the PI,

$$J = (X, u)\frac{1}{2}\int_0^2 u^2(t)\,dt$$

Find a set of necessary conditions for the optimal control. 7

3. a) Using the definition, determine the differential of the functional,

$$J(X) = \int_{t_0}^{t_f} \left[x_1^2(t) + x_1(t)x_2(t) + x_2^2(t)\,2\dot{x}_1(t)\dot{x}_2(t) \right] dt$$

Assume that the end points are specified.

b) Find the extremals for the functional,

$$J(X) = \int_0^{\pi/2} \left[\dot{x}_1^2(t) + 2x_1(t)x_2(t) + \dot{x}_2^2(t) \right] dt$$

with $x_1(0) = 0, x_1(\pi/2) = 1, x_2(0) = 0, x_2(\pi/2) = 1$.

c) Let $f(x) = -x_1 x_2$ and $g(x) = x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of $f(x)$ subject to the condition $g(x) = 0$. 3 + 3 + 8

4. a) Briefly discuss the different performance indices which are generally used for formulation of different optimal control problems.

   b) The auto-correlation function of a noise signal is given by,

   $R_x(\tau) = \sigma^2 \exp(-\beta|\tau|)$ where σ and β are two constants.

   Obtain the transfer function of the shaping filter that will convert a unity white noise signal to this particular noise signal. 7 + 7

5. a) Distinguish between a random variable and a random process. What do you mean by strict-sense stationary, wide-sense stationary and ergodicity of a random process ?

   b) Show how the mean and covariance of the state vector propagate through time when a linear dynamic system is excited by a zero-mean white noise input. 6 + 8

6. a) What do you mean by prediction, filtering and smoothing problems ? In what sense is the Kalman filter an optimal filter ?

   b) A linear discrete-time system is described by,

   $$x_k = x_{k-1} + w_{k-1}$$
   $$z_k = x_k + v_k$$

   where the process noise and the measurement noise are zero-mean white noises with intensities 1 and 2 respectively.

   Calculate the Kalman gain K_k and the estimation error covariance P_k for $k = 1$ and 2 assuming $P_0 = 10$. Also determine their steady state values.

c) Explain how colored process disturbance situation can be accommodated in a standard Kala man filter setting.

4 + 6 + 4

7. a) Derive the return difference inequality property of the infinite-time LQR controller and show that this controller yields a minimum gain margin of infinity and phase margin of 60° for a SISO minimum phase system.

b) Write a short note on Loop Transfer Recovery method.

9 + 5