Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE)/SEM-1/MMA-101/2010-11

2010-11

APPLIED MATHEMATICS FOR ELECTRICAL ENGINEERS

Time Allotted : 3 Hours　　　　　　　　　　*Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer *all* questions : 7×2

a) What are slack and surplus variables ? Explain.

b) Explain what do you understand by "assignment problem".

c) Define cross-correlation function and state any two of its properties.

d) The power spectral density of a random proces $\{ x(t) \}$ is given by

$$S_{xx}(w) = \pi, \quad |w| < 1$$
$$0, \quad \text{elsewhere}$$

Find its auto-correlation function.

e) If $A + B$ is a singular matrix and det A + det $B = 0$, find det $(A + B)$.

f) State the necessary and sufficient condition for the diagonalization of matrix.

g) State whether the following are *True* or *False* :

i) Assignment technique is essentially a minimization technique.

ii) In assignment problem if the final cost matrix contains more than one zero at independent positions then the problem will have a unique solution.

(Long Answer Type Questions)

Answer any *four* of the following. 4 × 14

2. a) State and prove Euler's Lagrange's equation.

b) Find the external to the functional

$$J[y] = \int_0^1 \left[2x + 3y + (y')^2 \right] dx,\ y(0) = y(1) = 1 .$$

3. a) State and prove Beltrami's identity. Hence find the extremal of the functional

$$J[y] = \frac{1}{\sqrt{2g}} \int_{x_1}^{x_2} \frac{\sqrt{1 + (y')^2}}{y_1 - y}\, dx,\ y(x_1) = y_1,\ y(x_2) = y_2 .$$

b) State Brachistochrone problem and solve it.

4. a) Define matrix norm.

   b) Prove that if $A \square V_{n \infty n}$ and $\| . \|_a : V_{n \infty n} \varnothing R$ defined by

   $$\|A\|_a = \sum_{i,j} |a_{ij}| \ \forall (a_{ij}) \square V_{n \infty n}, \text{ then } \|A\|_a \text{ is the}$$

   matrix norm on $V_{n \infty n}$

   [$V_{n \infty n}$: the set of all square matrix of order n which is a real vector space].

5. a) A paper mill will produce two grades of paper, namely X and Y. Owing to raw material restriction, it cannot produce more than 400 tons of grade X and 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0·2 and 0·4 hours to produce a ton of products X and Y respectively with corresponding profits of Rs. 200 and Rs. 500 per ton. Formulate the above as an LPP to maximize the profit and find the optimum product mix.

   b) Solve the following LPP by graphical method.

   Maximize $Z = 5x_1 + 7x_2$

   subject to $x_1 + x_2 \le 4$,

   $3x_1 + 8x_2 \le 24$,

   $10x_1 + 7x_2 \le 35$,

   $x_1, x_2 \ge 0$.

6. Use simplex method to solve the LPP

Maximize $Z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \leq 4$,

$x_1 - x_2 \leq 2$,

$x_1, x_2 \geq 0$.

7. Using the following cost matrix, determine the optimum job assignment and the cost of the assignment.

		Jobs				
		1	**2**	**3**	**4**	**5**
	A	10	3	3	2	8
	B	9	7	8	2	7
Machine	***C***	7	5	6	2	4
	D	3	5	8	2	4
	E	9	10	9	6	10

8. a) Show that the random process $X(t) = A\cos(wt + \theta)$ is wide-sense stationary if A and w are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

b) Calculate the power spectral density of a stationary random process for which the auto-correlation is $R_{xx}(t) = \sigma^2 e^{-\alpha|\tau|}$.