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CS/M.Tech (EE)/SEM-1/EEP-105/2009-10

2009

APPLIED LINEAR ALGEBRA IN ELECTRICAL ENGINEERING

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. $5 \times 14 = 70$

1. a) Show that the set of all 2×2 non-singular real matrices forms a group under matrix multiplication.

   b) Let H be a subgroup of a group (G, o) and R be a binary relation on G defined by $a\ R\ b \Leftrightarrow a^{-1}\, o\, b \in H$ for $a, b \in G$. Prove that R is an equivalence relation.

   c) Let $(G, *)$ and $\left(G', *' \right)$ be two groups and $f : G \rightarrow G'$ be a homomorphism. Prove that

      i) $f (e) = e'$ where e and e' are the identities of G and G' respectively.

      ii) $f\left(a^{-1}\right) = \left[f(a)\right]^{-1}\ \forall\ a \in G$.   5 + 4 + 5

2. a) Prove that a subgroup of index 2 is a normal subgroup.

   b) Show that $(Z_5, +, .)$ is an integral domain where Z_5 has usual meaning.

   c) Let (R, +, .) be a ring and S be a non-empty subset of R. Show that S is a subring of R iff $a + (-b) \in S$ and $a \cdot b \in S$ for every pair of elements $a, b \in S$, where $-b$ is the negative element of b. 4 + 5 + 5

3. a) Show that a set of vectors $\{\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n\}$ in a vector space V over a field F is linearly dependent iff one of the vectors can be expressed as a linear combination of the rest.

   b) If $V = W_1 \oplus W_2$ and W_1, W_2 are finite dimensional, show that V is finite dimensional and $\dim V = \dim W_1 + \dim W_2$. 7 + 7

4. a) Show that any linearly independent set of vectors in a finite dimensional vector space V over a field F is either a basis of V or can be extended to a basis of V.

   b) Given two linearly independent vectors (1, 0, 1, 0) and (0, –1, 1, 0) of R^4, find a basis for R^4 that includes these two vectors.

c) Find the coordinate vector of $\alpha = (3, 1, -4)$ relative to the basis $B = \{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ of R^3.

5 + 5 + 4

5. a) Define rank and nullity of a linear mapping.

Let V and W be two vector spaces over a field F and V is finite dimensional.

If $T : V \to W$ be a linear mapping, show that nullity of T + rank of T = dim V.

b) Let $T: R^3 \to R^2$ be a linear mapping defined by

$T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$

Find the matrix of T relative to the ordered bases

$\{f_1 = (1, 1, 1), f_2 = (1, 1, 0), f_3 = (1, 0, 0)\}$ of R^3

$\{g_1 = (1, 3), g_2 = (1, 4)\}$ of. (2 + 7) + 5

6. a) Let V and W be two vector spaces over the same field F with dim $V = m$ and dim $W = n$. Show that dim Hom $(V, W) = mn$.

b) Let V be an n dimensional vector space over F and $\{\alpha_1, \alpha_2, \ldots\ldots\ldots \alpha_n\}$ be a basis of V. Let $f_1, f_2, \ldots\ldots\ldots, f_n \in V^*$ be the linear functionals defined by

$$f_i(\alpha_j) = \delta_{ij} = 1 \text{ when } i = j$$
$$= 0 \text{ when } i \neq j$$

Prove that $\{f_1, f_2, \ldots\ldots\ldots f_n\}$ is a basis of V^*. 7 + 7

7. a) Find the dual basis of the following basis of $R^3 \{ (1, 0, 0), (0, 1, 0), (0, 0, 1) \}$.

b) Let $V(F)$ be an inner product space. Let $\alpha, \beta \in V$ and $a \in F$. Prove that $|\alpha . \beta| \le \|\alpha\| \|\beta\|$.

c) Use the Gram-Schmidt process of orthogonalisation to obtain an orthonormal basis for the Euclidean space $V_3(R)$, starting from the basis $\{ (1, 1, 1), (0, 1, 1), (0, 0, 1) \}$. 4 + 5 + 5

8. a) Solve the following system of equations by LU decomposition method :

$x + y + z = 1$

$4x + 3y - z = 6$

$3x + 5y + 3z = 4$

b) Solve the following system of equations by Jacobi's method correct to 3 decimal places :

$83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71.$ 7 + 7